

An Alternative to Generalized Linear Models with Application

Mangga A.M, Nwaosu S. C., Ahmed A

Abstract— In model building, forecasting is the ultimate goal since it assists in policy implementation and further research work. This paper aimed at establishing a robust quantile regression analysis. Appropriate model and parameters of the model with their statistical test for effective prediction was established. It has been established that quantile regression relative to ordinary least squares produce regression estimates that are more robust against outliers. From 5th to 99th percentile, results showed that, at 25th percentile the number of tillers and plant height are more significant with p-value of 0.044 and 0.001 at 0.05 level of significance compare to others. The generalized linear models considered showed insignificance with p-value 1.000 and 0.760 at 0.05 level of significance. However, quantile regression tells us what happened as we move from the smallest to the highest quantile in estimating the goodness of fit test for the model for proper forecasting also, the model has been established at 25th percentiles. (Yield = -2419.596 + 1.352(Tillers) + 45.322(Height)) with the best yield which is significant and the best amongst others.

Index Terms— Quantile, Model, Percentile, Tillers and Height.

I. INTRODUCTION

Quantile Regression (QR) models have provided a valuable tool in economics, finance, and statistics as a way of capturing heterogeneous effects of covariates on the outcome of interest, exposing a wide variety of forms of conditional heterogeneity under weak distributional assumptions. Importantly, Quantile Regression also provides a framework for robust inference. Applying quantile regression to count data presents logical and practical complications which are usually solved by artificially smoothing the discrete response variable through jittering. Other recent approaches include that of Congdon (2017), in which the asymmetric Laplace distribution is combined with a Poisson model in a Bayesian framework, and the model-based quantile regression of Padellini and Rue (2018), in which quantiles are mapped to the parameters of a generalized linear model identified by a continuous version of a valid count distribution. Tzavidis et al. (2015) proposed a semiparametric M-quantile approach for counts that extends the ideas of Cantoni and Ronchetti (2001) and Breckling and Chambers

(2001). These methods avoid jittering, but depend on a limited choice of predefined parametric models. [1]

Frumento and Bottai (2016, 2017) suggested using a fully parametric approach and reformulated model. Quantile regression is a type of regression analysis used in statistics and econometrics. Whereas the Method of Least Squares (MLS) results in estimates of the conditional mean of the response variable given certain values of the predictor variables $[E(y/x)]$, quantile regression aims at estimating either the conditional median or other quantiles of the response variable. Essentially, quantile regression is the extension of Linear Regression and we use it when the conditions of linear regressions are not applicable.

Quantile regression methods provide an alternative approach for robust inference. Rather than relying exclusively on a single measure of conditional central tendency, the quantile regression approach allows the investigator to explore a range of conditional quantile functions thereby exposing a variety of forms of conditional heterogeneity. [3].

Constant coefficient linear time series models have played an enormously successful role in statistics, and gradually various forms of random coefficient time series models, have also emerged as viable competitors in particular fields of application. One variant of the latter class of models, although perhaps not immediately recognizable as such, is the linear quantile regression model. This model has received considerable attention in the theoretical literature, and can be easily estimated with the quantile regression methods proposed in Koenker and Bassett (1978). Curiously, however, all of the theoretical work dealing with this model (that we are aware of) focuses exclusively on the i.i.d. innovation case that restricts the autoregressive coefficients to be independent of the specified quantiles. In this project work we seek to relax this restriction and consider linear quantile autoregression models whose autoregressive (slope) parameters may vary with quantiles $\tau \in [0, 1]$. We hope that these models might expand the modeling options for time series that display asymmetric dynamics or local persistency.

Considerable recent research effort has been devoted to modifications of traditional constant coefficient dynamic models to incorporate a variety of heterogeneous innovation effects. An important motivation for such modifications is the introduction of asymmetries into model dynamics. It is widely acknowledged that many important economic

Mangga A.M, Department of Mathematical Sciences, Abubakar Tafawa Balewa University, P. M. B. 0248, Bauchi, Nigeria.

Nwaosu S. C., Department of Mathematics/Statistics, University of Agriculture, Makurdi, Nigeria.

Ahmed A, 2Department of Mathematical Sciences, Abubakar Tafawa Balewa University, P. M. B. 0248, Bauchi, Nigeria.

variables may display asymmetric adjustment paths Enders and Granger (1998)). We believe that quantile regression methods can provide an alternative way to study asymmetric dynamics and local persistency in time series. We propose a quantile autoregression (QAR) model in which autoregressive coefficients may take distinct values over different quantiles

of the innovation process. We show that some forms of the model can exhibit unit-root-like tendencies or even temporarily explosive behavior, but occasional episodes of mean reversion are sufficient to ensure stationarity. The models lead to interesting new hypotheses and inference apparatus for time series, and data saddled with outliers [4].

II. MATERIALS AND METHODS

Quantile Regression Model

For better understanding, we start quantile regression from the basic idea of linear regression.

Apart from the mean, the lower and upper quantile are also important. A regression model does not capture the pattern of the situation. To better understand the quantile regression, we take a leave from linear regression. The quantile regression model estimates the potential differential effect of a covariate on various quantiles in the conditional distribution. For the linear regression model, we have

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i \quad \dots (1)$$

From (3.1) above, ϵ_{iis} identically, independently and normally distribute with mean zero and unknown variance σ^2 . Owing to the fact that the error is normally distributed with mean zero, the function $\beta_0 + \beta_1 x$ is fitted to the data Corresponding to conditional mean of y given by $E[y/x]$ and is always interpreted as mean in the population of y values corresponding to the fixed value of x . Given p to denote proportion, that is ($0 < p < 1$). Then the corresponding quantile regression for the equation (1) above is

$$Y_i = \beta_0^{(p)} + \beta_1^{(p)} x_i + \epsilon_i^{(p)} \quad \dots (2)$$

This indicates that the proportion of the population having scores below the quantile at p . The p^{th} conditional quantile ϵ_i is defined as

$$Q^{(p)}(y_i|x_i) = \beta_0^{(p)} + \beta_1^{(p)} x_i \quad \dots (3)$$

Equation (3) above represent the conditional p^{th} quantile which can be determine by the quantile specific parameters $\beta_0^{(p)}$ and $\beta_1^{(p)}$ which are specific value of the covariate x_i .

Again, Let Y be a random variable with a distribution function F_Y and π be a real number between 0 and 1. The π^{th} quantile of F_Y denoted as $q_y(\pi)$ is the solution to $F_Y(q) = \pi$ which is given as

$$q_y(\pi) = F_Y^{-1}(\pi) = \inf\{y: F_Y(y) \geq \pi\}$$

where $0 < \pi < 1$ is the quantile level.

The π^{th} quantile of F_Y can be obtained by minimizing the following function with respect to q

$$\pi \int_{y-q} |y-q| dF_y(y) + (1-\pi) \int_{y<q} |y-q| dF_y(y)$$

$$\pi \int_{y-q} (y-q) dF_y(y) - (1-\pi) \int_{y<q} (y-q) dF_y(y)$$

Applying the first order condition for minimization problem which is by taking its partial derivatives with respect to q and equate the result to zero

$$\{-\pi \int_{y>q} (y-q) dF_y(y) - [-(1-\pi)] \int_{y<q} (y-q) dF_y(y)\} = 0 \quad \dots (5)$$

$$-\pi \int_{y>q} (y-q) dF_y(y) + (1-\pi) \int_{y<q} (y-q) dF_y(y) = 0 \quad \dots (6)$$

Substituting in the limit we have (7) and opening the bracket will yield (8)

$$-\pi[1-F_y(q)] + (1-\pi)F_y(q) = 0 \quad \dots (7)$$

$$-\pi + \pi F_y(q) + F_y(q) - \pi F_y(q) = 0$$

$$-\pi + F_y(q) = 0$$

$$\pi = F_y(q) \quad \dots (8)$$

Equation (8) solution is the π th quantile of F_y

Conditional Quantile

Using Chung-Ming, (2007) model, suppose we have Y as a response and X is the p dimensional predictor. $F_y(Y|X=x) = P(Y \leq y|X=x)$ denote the conditional cumulative density function of Y given $X=x$ then, the π th

conditional quantile of Y can be denoted as $Q\pi(Y|X = x) = \inf\{y: Fy(y|x) \geq \pi\}$ furthermore, if the random variable y depend on x that is event y happening conditioning on another random variable x is $Fy|x(y)$, its π th quantile can be given as

$$Q_{y|x}(\pi) = F_{y|x}^{-1}(\pi) * Q_{y|x} \quad \dots (9)$$

Equation (9) is a function of X , solving it by minimization and at the same time applying the first other condition yield

$$\min_q [\pi \int_{y>q} |y - q| dF_{y|x}(y) + (1 - \pi) \int_{y<q} |y - q| dF_{y|x}(y)] \quad \dots (10)$$

$Q_{Y|x}(0.5)$ is the conditional median which represents the center is the point of symmetry of $Q_{Y|x}$. If π is close to zero $Q_{Y|x}(\pi)$ is called the left tail of $F_{Y|x}$. Also if $Q_{Y|x}(\pi)$ is a linear function $X'\beta$, (when q is substitute with $X'\beta$) unknown up to the parameter vector β then equation (10) becomes

$$\min_q [\pi \int_{x>X'\beta} |x - X'\beta| dF_{x|y}(x) + (1 - \pi) \int_{x<X'\beta} |x - X'\beta| dF_{x|y}(x)] \quad \dots (11)$$

Using the same principle as (8) above, we have

$$Q_{Y|x}(\pi) = X'\beta_{\pi}$$

the solution is denoted as β_{π} which is the π th conditional quartile.

Parameters Estimation

Before estimating the parameters, let first of all look at ordinary least square. In OLS, we minimize the sum of squares of the error (the error term) and thereafter find the optional value of β_0 and β_1 that is

$$OLS = \min \sum_{i=1}^n [y_i - (\beta_0 + \beta_1 x_i)]^2 \dots (12)$$

For the Quantile regression, we replace the square with absolute therefore its minimizes the absolute least deviation (LAD)

$$LAD = \min \sum_{i=1}^n |y_i - (\beta_0 + \beta_1 x_i)| \dots (13)$$

The error term $e_i = y_i - (\beta_0 + \beta_1 x_i)$ and hence the Quantile regression denoted by τ which is an extension least absolute deviation (LAD) can be expressed as

$$\tau = \sum_{i=1}^n \{Q |e_i| + (1 - Q) |e_i|\} \quad \dots (14)$$

replacing the value of $e_i = y_i - (\beta_0 + \beta_1 x_i)$ in equation (14) and we have

$$\tau = \sum_{i=1}^n \{Q |y_i - (\beta_0 + \beta_1 x_i)| + (1 - Q) |y_i - (\beta_0 + \beta_1 x_i)|\}$$

$$\tau = \sum_{i=1}^n Q |y_i - (\beta_0 + \beta_1 x_i)| + \sum_{i=1}^n (1 - Q) |y_i - (\beta_0 + \beta_1 x_i)| \quad \dots (15)$$

it convenience to write (15) as

$$\tau = \min_{\beta_0, \beta_1} \sum_{i: y_i \geq \beta_0 + \beta_1 x_i} Q |y_i - (\beta_0 + \beta_1 x_i)| + \min_{\beta_0, \beta_1} \sum_{i: y_i \leq \beta_0 + \beta_1 x_i} (1-Q) |y_i - (\beta_0 + \beta_1 x_i)| \quad \dots (16)$$

For OLS, we find the β_0 and β_1 by differentiation, maximum likelihood estimation but in quantile regression, we make use of linear programming (simplex algorithm, integer point algorithm or simplex algorithm).

We formulate quantile regression problem in a way analogous to the formulation of least square (conditional mean) regression. Let Y be a random variable with some distribution function and a sample $y_i, i=1,2,\dots,n$. The median of the set of sample y_i can be defined as the solution of the minimization problem.

$$\min_{\beta} \sum_i^n |y_i - \beta|, \beta \in R \quad \dots (17)$$

Estimating the value of β in (17) above, that is θ^{th} sample of y_i then we have

$$\min_{\beta} \{ \sum_{y \geq \beta} Q |y_i - \beta| + \sum_{y < \beta} (1-Q) |y_i - \beta| \}, \beta \in R \quad \dots (18)$$

Where Q is penalty imposed on under prediction and $1-Q$ is penalty imposed on over prediction. From linear regression model, let consider a set of random variables $y_i, i \in [1, n], n \in \mathbb{N}$ that are paired with a set of $X = \{x_i\}, i=1,2,\dots,n$ and y_i is a realization of Y and hence we have

$$\min_{\beta_0, \beta_1} \sum_{i \in \{i: y_i \geq \beta_0 + \beta_1 x_i\}} Q |y_i - (\beta_0 + \beta_1 x_i)| + \min_{\beta_0, \beta_1} \sum_{i \in \{i: y_i < \beta_0 + \beta_1 x_i\}} (1-Q) |y_i - (\beta_0 + \beta_1 x_i)| \quad \dots (19)$$

Since β can be β_0, β_1 and therefore $\beta \in R$ becomes $\beta_0, \beta_1 \in R$. Converting the (19) to linear programming problem, we introduce a non-negative variable s_i and r_i for which the following equation holds

$$y_i - (\beta_0 + \beta_1 x_i) + s_i = 0, i \in \{i: y_i \geq \beta_0 + \beta_1 x_i\} \quad \dots (20)$$

$$s_i = 0, i \notin \{i: y_i \geq \beta_0 + \beta_1 x_i\}$$

$$(\beta_0 + \beta_1 x_i) - y_i + r_i = 0, i \in \{i: y_i < \beta_0 + \beta_1 x_i\} \quad \dots (21)$$

$$r_i = 0, i \in \{i: y_i < \beta_0 + \beta_1 x_i\}$$

Since s_i and $r_i > 0$ on complementary sets. We can re-write (20) and (21)

$$y_i - (\beta_0 + \beta_1 x_i) + s_i - r_i = 0 \quad \dots (22)$$

$$s_i \geq 0, r_i \geq 0, i \in [1, n] \quad \dots (23)$$

Then (19) can be express as

$$\min_{s_i, r_i, \beta_0, \beta_1} \{ \sum_{i \in \{i: y_i \geq \beta_0 + \beta_1 x_i\}} Q s_i + \sum_{i \in \{i: y_i < \beta_0 + \beta_1 x_i\}} (1-Q) r_i \} \quad \dots (24)$$

Since $s_i \geq 0, r_i \geq 0$ then minimization function of (24) above becomes

$$\min_{s_i, r_i, \beta_0, \beta_1} \left\{ \sum_{i=1}^n Qs_i + \sum_{i=1}^n (1-Q)r_i \right\} \dots (25)$$

Equation (22), (23) and (25) are the linear programming formulation of the quantile regression of (19) above. In summary, the estimation of parameters in quantile regression is done via linear programming.

III. RESULTS AND DISCUSSION

Table Summary of Quantile Regression

| Yield | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|--------------|-----------|-----------|-------|-------|----------------------|-----------|
| 0.05.HEIGHT | 26.82915 | 10.95797 | 2.45 | 0.018 | 4.771909 | 48.8864 |
| TILLERS | -.3909365 | .9412576 | -0.42 | 0.680 | -2.28559 | 1.503717 |
| _CONS | -1281.001 | 674.3538 | -1.90 | 0.064 | -2638.405 | 76.40236 |
| 0.10 HEIGHT | 28.42516 | 11.71113 | 2.43 | 0.019 | 4.851876 | 51.99845 |
| TILLERS | .3078091 | .977165 | 0.32 | 0.754 | -1.659122 | 2.27474 |
| _CONS | -1434.758 | 715.5499 | -2.01 | 0.051 | -2875.085 | 5.569423 |
| 0.15 HEIGHT | 26.15 | 12.83133 | 2.04 | 0.047 | .3218785 | 51.97811 |
| TILLERS | 1.449999 | .8440589 | 1.72 | 0.093 | -.249003 | 3.149002 |
| _CONS | -1393.05 | 748.4139 | -1.86 | 0.069 | -2899.529 | 113.4293 |
| 0.20 HEIGHT | 38.93838 | 13.3661 | 2.91 | 0.006 | 12.03381 | 65.84295 |
| TILLERS | 1.416289 | .7238217 | 1.96 | 0.056 | -.0406884 | 2.873266 |
| _CONS | -2080.622 | 756.3746 | -2.75 | 0.008 | -3603.125 | -558.1194 |
| 0.25. HEIGHT | 45.32166 | 12.67594 | 3.58 | 0.001 | 19.80633 | 70.83699 |
| TILLERS | 1.351982 | .6517332 | 2.07 | 0.044 | .0401111 | 2.663853 |
| _CONS | -2419.596 | 706.9043 | -3.42 | 0.001 | -3842.521 | -996.6714 |
| 0.30. HEIGHT | 54.33097 | 11.80222 | 4.60 | 0.000 | 30.57434 | 78.0876 |
| TILLERS | .8941423 | .8731224 | 1.02 | 0.311 | -.8633619 | 2.651647 |
| _CONS | -2864.586 | 658.998 | -4.35 | 0.000 | -4191.081 | -1538.092 |
| 0.35 HEIGHT | 47.49151 | 11.00345 | 4.32 | 0.000 | 25.34271 | 69.6403 |
| TILLERS | .4441779 | .9960077 | 0.45 | 0.658 | -1.560682 | 2.449037 |
| _CONS | -2364.226 | 636.2711 | -3.72 | 0.001 | -3644.973 | -1083.479 |
| 0.40 HEIGHT | 46.71641 | 11.45156 | 4.08 | 0.000 | 23.66561 | 69.7672 |
| TILLERS | .9175528 | 1.161073 | 0.79 | 0.433 | -1.419565 | 3.254671 |
| _CONS | -2320.985 | 669.2884 | -3.47 | 0.001 | -3668.192 | -973.7769 |
| 0.45 HEIGHT | 52.90792 | 11.87167 | 4.46 | 0.000 | 29.01148 | 76.80436 |
| TILLERS | .791709 | 1.418251 | 0.56 | 0.579 | -2.063081 | 3.646499 |
| _CONS | -2629.956 | 719.3343 | -3.66 | 0.001 | -4077.901 | -1182.011 |
| 0.50 HEIGHT | 53.96568 | 11.90502 | 4.53 | 0.000 | 30.00212 | 77.92924 |
| TILLERS | .7645777 | 1.750539 | 0.44 | 0.664 | -2.759075 | 4.288231 |
| _CONS | -2681.051 | 745.561 | -3.60 | 0.001 | -4181.788 | -1180.315 |
| 0.55 HEIGHT | 50.95909 | 13.0534 | 3.90 | 0.000 | 24.68397 | 77.23422 |
| TILLERS | .2454546 | 2.039178 | 0.12 | 0.905 | -3.859198 | 4.350107 |
| _CONS | -2356.946 | 824.8908 | -2.86 | 0.006 | -4017.365 | -696.5264 |
| 0.60 HEIGHT | 45.91223 | 14.40138 | 3.19 | 0.003 | 16.92376 | 74.90071 |
| TILLERS | -.6259455 | 2.630528 | -0.24 | 0.813 | -5.920924 | 4.669033 |
| _CONS | -1812.901 | 950.4585 | -1.91 | 0.063 | -3726.075 | 100.2722 |
| 0.65 HEIGHT | 48.35139 | 14.48095 | 3.34 | 0.002 | 19.20276 | 77.50003 |
| TILLERS | -.7718782 | 2.710668 | -0.28 | 0.777 | -6.228169 | 4.684413 |
| _CONS | -1905.715 | 965.1009 | -1.97 | 0.054 | -3848.362 | 36.93283 |
| 0.70 HEIGHT | 49.12381 | 16.04288 | 3.06 | 0.004 | 16.83117 | 81.41646 |
| TILLERS | -1.22 | 3.406536 | -0.36 | 0.722 | -8.077001 | 5.637002 |
| _CONS | -1814.533 | 1051.535 | -1.73 | 0.091 | -3931.164 | 302.0966 |

```

0.75 HEIGHT | 44.68595 18.02229 2.48 0.017 8.408963 80.96294
    TILLERS | -1.643614 4.110853 -0.40 0.691 -9.918332 6.631104
    _CONS | -1438.929 1215.085 -1.18 0.242 -3884.769 1006.911
0.80  HEIGHT | 46.99695 18.95242 2.48 0.017 8.847708 85.14618
    TILLERS | 2.018369 4.573416 0.44 0.661 -7.18744 11.22418
    _CONS | -1909.834 1339.197 -1.43 0.161 -4605.498 785.8297
0.85  HEIGHT | 48.55745 19.45285 2.50 0.016 9.400898 87.714
    TILLERS | 6.913523 5.021539 1.38 0.175 -3.19431 17.02136
    _CONS | -2421.605 1358.815 -1.78 0.081 -5156.759 313.5478
0.90  HEIGHT | 31.27261 18.56168 1.68 0.099 -6.090104 68.63533
    TILLERS | 5.18504 5.012381 1.03 0.306 -4.90436 15.27444
    _CONS | -952.3943 1300.671 -0.73 0.468 -3570.51 1665.721
0.95  HEIGHT | 37.65052 19.71045 1.91 0.062 -2.024554 77.32559
    TILLERS | 1.540521 4.698095 0.33 0.744 -7.916253 10.9973
    _CONS | -766.5238 1170.506 -0.65 0.516 -3122.63 1589.582
0.99  HEIGHT | 41.86546 20.02067 2.09 0.042 1.565931 82.16499
    TILLERS | -8.680157 3.855182 -0.23 0.823 -8.628094 6.892062
    _CONS | -643.6884 1081.384 -0.60 0.555 -2820.401 1533.024
    
```

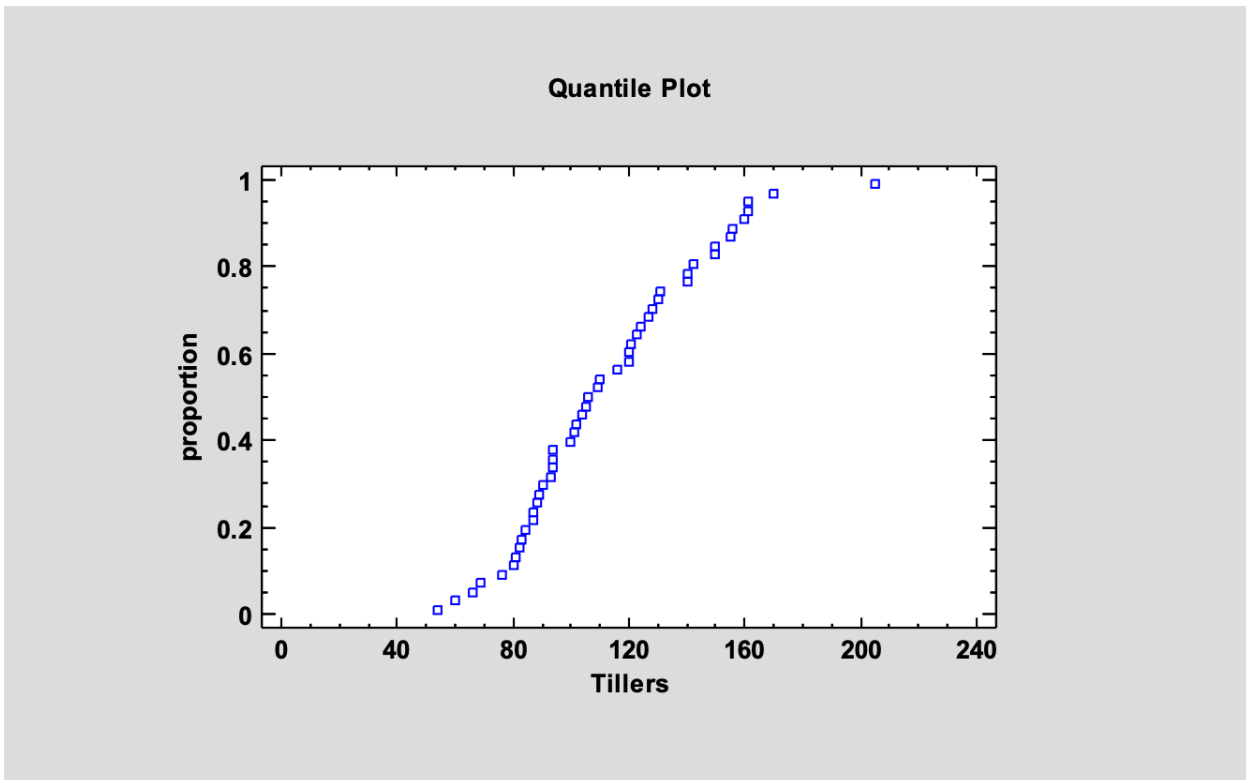


Fig. 4.1: Quantile Plot showing the distribution of the number of Tillers

The above procedure creates a plot that shows the proportion of data below each observed value of the number of Tillers.

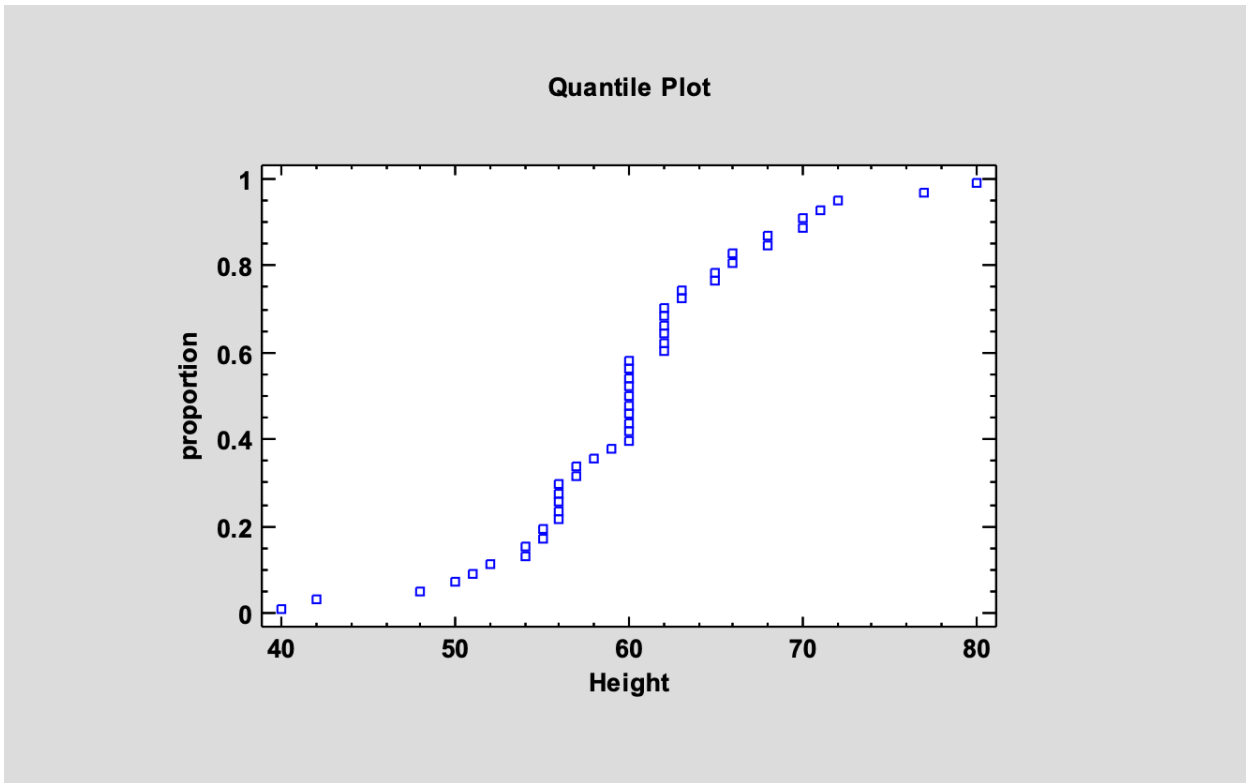


Fig. 4.2: Quantile Plot showing the distribution of Height (cm)

The above procedure creates a plot that shows the proportion of data below each observed value of the number of Tillers.

Discussion of Results

The quantiles refer to the general case of dividing a dataset or population into quarters. In this work we made use of twenty (20) quantiles, namely: 5th quantile, 10th quantile, 15th quantile, 20th quantile, 25th quantile, 30th quantile, 35th quantile, 40th quantile, 45th quantile, 50th quantile (Median), 55th quantile, 60th quantile, 65th quantile, 70th quantile, 75th quantile, 80th quantile, 85th quantile, 90th quantile, 95th quantile and 99th quantile using STATA Version 15 Statistical software. The research modelled the Grain Yield (kg/ha) as a function of the number of Tillers and the height (cm) of the plant.

From the summary table above, the result of 5th quantile showed that the number of tillers is insignificant with p-value of 0.680 while plant height is significant with p-value of 0.018 at 0.05 level of significance. The result of 10th quantile showed that the number of tillers is insignificant with p-value of 0.754 while plant height is significant with p-value of 0.019 at 0.05 level of significance. The result of 15th quantile showed that the number of tillers is insignificant with p-value of 0.093 while plant height is significant with p-value of 0.047 at 0.05 level of significance.

The result of 20th quantile showed that the number of tillers and plant height are insignificant with p-value of 0.056 and 0.006 respectively at 0.05 level of significance. The result of 25th quantile showed that the number of tillers and plant

height are significant with p-value of 0.044 and 0.001 respectively at 0.05 level of significance. The result of 30th quantile showed that the number of tillers is insignificant with p-value of 0.311 while plant height is significant with p-value of 0.000 at 0.05 level of significance. The result of 35th quantile showed that the number of tillers is insignificant with p-value of 0.658 while plant height is significant with p-value of 0.000 at 0.05 level of significance.

The result of 40th quantile showed that the number of tillers is insignificant with p-value of 0.433 while plant height is significant with p-value of 0.000 at 0.05 level of significance. The result of 45th quantile showed that the number of tillers is insignificant with p-value of 0.579 while plant height is significant with p-value of 0.000 at 0.05 level of significance. The median Regression result showed that the number of tillers is insignificant with p-value of 0.664 while plant height is significant with p-value of 0.000 at 0.05 level of significance. The result of 55th quantile showed that the number of tillers is insignificant with p-value of 0.905 while plant height is significant with p-value of 0.000 at 0.05 level of significance. The result of 60th quantile showed that the number of tillers is insignificant with p-value of 0.813 while plant height is significant with p-value of 0.003 at 0.05 level of significance. The result 65th quantile showed that the number of tillers is insignificant with p-value of 0.777 while plant height is significant with p-value of 0.002 at 0.05 level of significance.

The result of the 70th quantile showed that the number of tillers is insignificant with p-value of 0.722 while plant height is significant with p-value of 0.004 at 0.05 level of

significance. The result of the 75th quantile showed that the number of tillers is insignificant with p-value of 0.691 while plant height is significant with p-value of 0.017 at 0.05 level of significance. The result of 80th quantile showed that the number of tillers is insignificant with p-value of 0.661 while plant height is significant with p-value of 0.017 at 0.05 level of significance. The result 85th quantile showed that the number of tillers is insignificant with p-value of 0.175 while plant height is significant with p-value of 0.016 at 0.05 level of significance. The result of the 90th quantile showed that the

number of tillers is insignificant with p-value of 0.306 while plant height is significant with p-value of 0.099 at 0.05 level of significance. The result of the 95th quantile showed that the number of tillers and plant height are insignificant with p-value of 0.744 and 0.062 respectively at 0.05 level of significance. The result of the 99th quantile showed that the number of tillers and plant height are insignificant with p-value of 0.823 and 0.042 respectively at 0.05 level of significance.

Analysis of Poisson Regression Model

Generalized linear models No. of obs = 49
 Optimization : ML Residual df = 46

Variance function: V(u) = u [Poisson]
 Link function : g(u) = ln(u) [Log]

AIC = 8.325254
 Log likelihood = -200.9687141 BIC = -179.0237

| | Observed | Bstrap * | | | | |
|-----------|--------------|-----------|-------|-------|----------------------|----------|
| Yield | Coef. | Std. Err. | z | P> z | [95% Conf. Interval] | |
| Height | 3.86e-15 | 2.03e-10 | 0.00 | 1.000 | -3.97e-10 | 3.97e-10 |
| Tillers | 7.12e-16 | 3.55e-11 | 0.00 | 1.000 | -6.96e-11 | 6.96e-11 |
| _cons | -2.74e-13 | 1.30e-08 | -0.00 | 1.000 | -2.54e-08 | 2.54e-08 |
| ln(Yield) | 1 (exposure) | | | | | |

Analysis of Gaussian Regression Model

Generalized linear models No. of obs= 49
 Optimization : ML Residual df = 46

Variance function: V(u) = 1 [Gaussian]
 Link function : g(u) = u [Identity]

AIC = 15.14927
 Log likelihood = -368.1570605 BIC = 9633368

| | Observed | Bstrap * | | | | |
|-----------|--------------|-----------|-------|-------|----------------------|-----------|
| Yield | Coef. | Std. Err. | z | P> z | [95% Conf. Interval] | |
| Height | 42.69765 | 10.21391 | 4.18 | 0.000 | 22.67876 | 62.71655 |
| Tillers | .4219653 | 1.380386 | 0.31 | 0.760 | -2.283541 | 3.127472 |
| _cons | -1820.388 | 647.7526 | -2.81 | 0.005 | -3089.96 | -550.8161 |
| ln(Yield) | 1 (exposure) | | | | | |

In generalized linear models

Poisson regression model, the result showed that the number of tiller and plant height are insignificant with p-value of 1.000 and 1.000 respectively at 0.05 level of insignificance.

For Gaussian regression model, the result showed that the number of tiller is insignificant and plant height is significant with p-value of 0.000 and 0.760 respectively at 0.05 level of significance.

IV. CONCLUSION

In this research work, we have demonstrated the potential use of quantile regression in dealing with rainfed barley observation nursery. Using real dataset, the result showed in table 4.5 above, that the number of tillers and plant height are significant with p-value of 0.044 and 0.001 respectively at 0.05 level of significance. Meanwhile the generalized linear model, the poisson regression model and Gaussian regression model is not significant. Quantile regression is a robust regression to outliers compared to generalized linear regression models. Traditional means regression models like Generalized Linear Model (GLM) are not able to capture the entire distribution of the data.

However, from quantile 5 to 99, the quantile regression tells us what happened as we move from the smallest to the highest quantile. In estimating the goodness of fit test for the model for proper forecasting. The model has been established in equation 25th Quantile with the best yield which is significant and the best amongst others. Quantiles regression will be able to provide not just a single value for estimation but for numerous quantiles.

REFERENCES

- [1] [André F. B. M., Josmar Mazucheli, Marcelo Bourguignon](#) (2020). A parametric quantile regression approach for modelling zero-or-one inflated double bounded data. *Biometrical Journal*.
- [2] Aznarte, J. L. (2017). Probabilistic forecasting for extreme no 2 pollution episodes. *Environ. Pollut.* 229, 321–328
- [3] Baione F. & Biancalana D. (2019). An individual risk model for premium calculation based on quantile: a comparison between generalized linear models and quantile regression. *North American Actuarial Journal* 23, 573–590. [[Taylor & Francis Online](#)], [[Web of Science @](#)], [[Google Scholar](#)]
- [4] Baione F., Biancalana D. & De Angelis P. (2019). A quantile regression approach for the analysis of the diversification in non-life premium risk. *Soft Computing* 24, 8523–8534. Doi: 10.1007/s00500-019-04291-x. [[Crossref](#)], [[Web of Science @](#)], [[Google Scholar](#)]
- [5] Bai, L., Wang, J., Ma, X. & Haiyan, L. (2018). Air pollution forecasts: An overview. *Int. J. Environ. Res. Public Health* 15(4).
- [6] Baur, D., Saisana, M. & Schulze, N. (2004). Modelling the effects of meteorological variables on ozone concentration: A quantile regression approach. *Atmos. Environ.* 38(28), 4689–4699.
- [7] Bergmeir, C., Hyndman, R.J. & Koo, B. (2018). A note on the validity of cross-validation for evaluating autoregressive time series prediction. *Comput. Stat. Data Anal.* 120, 70–83.
- [8] Biancalana D. (2017). Unapproccio quantile regression per la tariffazione degli anni, basatosu un modello a due parti. Ph.D. Thesis. <https://iris.uniroma1.it/retrieve/handle/11573/1209293/94477/7/Tesi%20dottorato%20Biancalana.pdf>. [[Google Scholar](#)]
- [9] Brian, S.C. (2003). Quantile Regression Models of Animal Habitat relationships. Ph.D. Thesis Colorado State University Fort Collins, Colorado Spring. 194pp
- [10] Dong A., Chan J. & Peters G. (2015). Risk margin quantile function via parametric and non-parametric Bayesian approaches. *ASTIN Bulletin* 45(3), 503–550. doi: 10.1017/asb.2015.8. [[Crossref](#)], [[Web of Science @](#)], [[Google Scholar](#)]
- [11] [Fabio Baione & Davide Biancalana](#) (2020). An application of parametric quantile regression to extend the two-stage quantile regression for ratemaking. *Scandinavian actuarial journal* (2020) vol 2021 (6).
- [12] Frees E. W. (2010). *Regression modeling with actuarial and financial applications*. New York: Cambridge University Press. [[Google Scholar](#)]
- [13] Frees E. W., Jin X. & Lin X. (2013). Actuarial applications of multivariate two-part regression models. *Annals of Actuarial Science* 7, 258–287. [[Crossref](#)], [[Google Scholar](#)]
- [14] Frumento P. (2017). QRCM: quantile regression coefficients modeling. R package version 2.1. <https://cran.r-project.org/package=qrcm>. [[Google Scholar](#)]
- [15] Frumento P. & Bottai M. (2016). Parametric modeling of quantile regression coefficient functions. *Biom* 72, 74–84. doi:10.1111/biom.12410. [[Crossref](#)], [[Google Scholar](#)]
- [16] Frumento P. & Bottai M. (2017). Parametric modeling of quantile regression coefficient functions with censored and truncated data. *Biometrics* 73(4), 1179–1188. doi: 10.1111/biom.12675 [[Crossref](#)], [[PubMed](#)], [[Web of Science @](#)], [[Google Scholar](#)]
- [17] Fuzi M. F. M., Jemain A. A. & Noriszura I. (2016). Bayesian quantile regression model for claim count data. *Insurance, Mathematics & Economics* 66, 124–137. [[Crossref](#)], [[Web of Science @](#)], [[Google Scholar](#)]
- [18] Heras A., Moreno I. & Vilar-Zanón J. L. (2018). An application of two-stage quantile regression to insurance ratemaking. *Scandinavian Actuarial Journal* 9, 753–769. [[Taylor & Francis Online](#)], [[Google Scholar](#)]
- [19] Hong, T. et al. (2016). Probabilistic energy forecasting: Global energy forecasting competition 2014 and beyond. *Int. J. Forecast.* 32(3), 896–913.
- [20] Hothorn, T., Kneib, T. & Bühlmann, P. (2014). Conditional transformation models. *J. R. Stat. Soc. B* 76(1), 3–27.
- [21] Jang, Y., Kim, J. H., Lee, H., Lee, K. & Ahn, S. A. (2018). A quantile regression approach to explain the relationship of fatigue and cortisol, cytokine among Koreans with Hepatitis b. *Sci. Rep.* 8(1), 16434.
- [22] Ke, G. et al. (2017). LightGBM: A Highly Efficient Gradient Boosting Decision Tree. *Adv. Neural Inf. Process. Syst.* 30, 3146–3154.
- [23] Koenker R. (2005). *Quantile regression*. Econometric Society Monograph Series, Vol. 38. Cambridge: Cambridge University Press. [[Crossref](#)], [[Google Scholar](#)]
- [24] Koenker R. (2015). Quantreg: quantile regression. R package version 5.19. <http://CRAN.R-project.org/package=quantreg>. [[Google Scholar](#)]
- [25] Lebotsa, M. E. et al. (2018). Short-term electricity demand forecasting using partially linear additive quantile regression with an application to the unit commitment problem. *Appl. Energy* 222, 104–118.
- [26] Mangalova, E. & Shesterneva, O. (2016). K-nearest neighbors for gefcom2014 probabilistic wind power forecasting. *Int. J. Forecast.* 32(3), 1067–1073.
- [27] Martínez-Silva, I., Roca-Pardiñas, J. & Ordóñez, C. (2016). Forecasting SO₂ pollution incidents by means of quantile curves based on additive models. *Environmetrics* 27(3), 147–157.