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Three-Dimensional Pathway for Disjoined Chess Board with Vector Directives

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***Abstract*—This paper, bring out the history of the game of chess that can be traced back as far as the 6th Century. However, the chess game started in the Northern part of India and extended to Persia from there, it spread to the entire Asian Continent. In-addition, we reviewed some relevant literature with current state of knowledge appurtenance to the higher dimensional rook movement on a chess board within the forbidden area were studied. We also reviewed some basic techniques that are useful in studying and analyzing the rook movements on a three-dimensional chess board. However, we showed that, the sum of the rook movement is the product of the corresponding direction of the angle between the two vectors that give the three- dimensional structure. In-addition, we showed how a board B can be connected to a three-dimensional disjoined rook boards with forbidden squares and the t non-attacking rook movements to generate a rook function. Furthermore, some combinatorial problems were solved by applying generating functions to the rook movement with forbidden squares for the t non-attacking rook movements.**

***Index Terms*—Chess movements, combinatorial structures,Disjoined chess board in three-dimensions, generating functions, three-dimensional structures**

I. INTRODUCTION

The history of the game of chess can be traced back as far as the 6th Century about 1500 years ago. It started in India especially in the Northern part of India and extended to Persia from there, it spread to the entire Asian Continent. In the 10th Century, the chess game made a huge impact by spreading to Arabian Islamic empire to Europe. Until the end of the 15th Century, the game of chess changed many times and survived sanctions by Christian churches as well as complete prohibitions from time to time. Then, by 1880, the chess game progressively developed into today's modern game. In the year 1880 – 1950, the romantic era of chess started where chess players relied on sacrifices tactics and extremely dynamic playing techniques. The first official World Championship of chess game was hosted in 1886 where Wilhelm Steinitz became the first World official champion of chess game. The 20th century revolutionized chess game with the invention of chess engines and databases. In 1997, the World chess champion Kasparov loss of six (6) games match to the IBM's computer Deep Blue. Then, the online chess game and website of the chess were invented and became so popular between the years 2007 – 2018. In the year 2007, chess website (chess.com) was launched followed by the Li-chess and chess 24's website in the year 2010 and 2014 respectively.

However, chess game is known as the World over played game with numerous fans but its origins and roots are not clear and it is highly debatable. There are variety of legends, plain guesses, and stories about chess game starting from a dispute over where the game came from and ending with when chess game began. Most people agreed that there was not just one individual that invented and established the game because the playing techniques are too complex with all its rules and concepts for an individual mind to have created. Then, until Wilhelm Steinitz became the first World official chess champion in 1886, the chess game was in a steady flux. One ancient legend "a tyrannical Indian King" called Shahram and a wise man in his kingdom was the first person that invented the chess game. The wise man wanted to convince Shahram of the important of every resident in his Kingdom. So, he invented a game to represent his kingdom consisting of the king himself, his queen, rooks, bishops, knights, and pawns all of which were very important. The king was surprised and liked the game very well and understood that the game was just a real life where everyone is important. Then, the king ordered everybody in his kingdom to play the game (chess). Shahram appreciated the wise man and from there, the people studied and started playing the chess game.

Furthermore, there are techniques that guide the chess board player which are as follows;

1. The king moves exactly one square vertical, horizontal or diagonal. The king is allowed to make a special movement called the castling in at most once in every game.
2. The queen moves any number of vacant squares of the chess board i.e. horizontally, vertically or diagonally.
3. The rook moves any number of vacant squares on the chessboard vertically or horizontally and also its move during castling.
4. The bishop moves any number of vacant squares on the chess board in a diagonal direction.
5. The knight moves one square along any file or rank and then to an angle on a chess board. Also, the movement of the knight can be viewed as a "T" or "L" (i.e. the representative symbol for the movement of the knight) laid out at any horizontal or vertical angle on a chess board.
6. The Pawn moves forward one square if the square is unoccupied. If it has not moved yet, the pawn has the option of moving two squares forward on a chessboard provided that both squares in the front of the pawn are unoccupied. It is very important to note that the pawn cannot move backward and it is the only piece that has the right to capture diagonally

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from how they move. They can capture an enemy piece on the chess board when there are two spaces adjacent to the space in front of them (i.e. two squares diagonally in their front) but cannot move to these spaces if they are empty or vacant. Also, the pawn is involved in the two special movements called the en-passant (i.e. when the pawn is moved two squares on its initial movement).

However, the problem of counting objects where the arrangements have restrictions in some of the positions in which they can be placed has been addressed (Laisin, 2018; Laisin and Ndubuisi, 2017). Rook theory has a long interesting history arising from permutation problems with restricted positions (Laisin, 2018; Michaels, 2013). Herckman, 2006; Jay & Haglund, 2000), with further developments on Ferrers boards which was first developed by Foata and Schutzenderger in 1970 with details on characterization of rook equivalence through bijective proofs (Goldman, Joichi and White 1977). However, work with Ferrers board and this rook polynomial have led to modeling of some theorems of binomial (Goldman, Joichi, Reiner, and White, 1977) with connections to chromatic polynomials (Laisin and Ndubuisi, 2017; Goldman, Joichi, and White,1977), Orthogonal polynomial (Goldman, Joichi, and White,1977) hyper geometric series (Haglund, 1996) and so many other large literatures on Ferrers board. While, Jay and Haglund in 2000 generalized a classic notion of rook polynomials by placing non-taking rooks on a Ferrers board where rooks are placed in the columns of the board moving from left to right as new rows are created.

Also, studies by many authors/Scholars have proved that rook polynomials of any board can be computed recursively using cell decomposition techniques of Riordan. Similarly, many authors have studied the generalization of rook polynomials on boards in higher dimensions (Laisin, Okeke and Chukwuma, 2020; Sangeetha and Jayalalitha in 2017; Nicholas, and Feryal, 2009; White., Goldman, and Joichi, 1975). In-addition, the theory of rook polynomials focuses on the t non – attacking placement of rooks in a more general case. Sangeetha and Jayalalitha in 2017 classified the quadratic rook polynomials for a generalized three-dimensional board. In higher dimension the rooks attack along with hyper plane which corresponds to layers of cell (block) with one fixed coordinate. Sangeetha and Jayalalitha in 2017 concluded their study by enumerating all the possible quadratic rook polynomials of a generalized three-dimensional board for the values of r_1 ranging from $r_1 = 3$ to $r_1 = 10$.

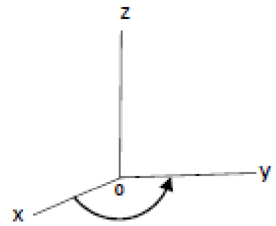
Furthermore, rook polynomials in higher dimensions by Nicholas and Feryal, 2009 was to generalize the properties and theorems of three-dimensional boards into a higher dimensional board which was first introduced by Zindle in 2007 while Nicholas and Feryal in 2009 included a Maple program which calculates the number of rooks in a given three-dimensional board using this generalization. In-addition, Laisin, Okeke and Chukwuma in 2020 showed that, the three-dimensional angle for rook movement within the restricted area is its cosines of the angle generated by the vectors on the chess board.

Now, we shall be focusing on a chess board in three dimensions to show that, the sum of the rook movement is the product of the corresponding direction of the angle between the two vectors that give the three- dimensional structure. In-addition, we shall generate a rook function for a three-dimensional rook boards with forbidden squares that is generated by the vector's directives.

II. BASIC DEFINITIONS

II.0.1 DEFINITION

The axes of reference are defined by the right-hand rule. OX, OY and OZ. Thus, a right-handed corkscrew action along the positive direction of OZ.



Similarly, rotation from OY to OZ gives a right-handed corkscrew action along the positive direction of OX

II.0.2 DEFINITION

Consider the diagram. If the vector $OP = r = ai + bj + ck$ and is defined by its components such that; a is along OX, b is along OY and c is along OZ.

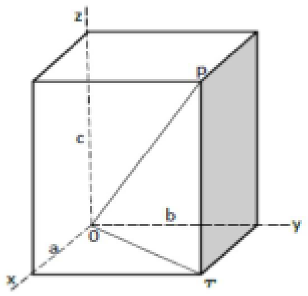


fig. 0.2

Now, consider fig. 0.2 we have that, the unit vector are as follows; **i** is in the OX direction, **j** is in the OY direction and **k** is in the OZ direction. Then, $0P = ai + bj + ck$ also$0T^2 = a^2 + b^2$

Implies $OP^2 = a^2 + b^2 + c^2$ so, if $r = ai + bj + ck$ then$|r| = r = \sqrt{a^2 + b^2 + c^2}$

Theorem II.1.1

If the movement on a three-dimensional disjoined chess boards is a rook movement, then the angle between the vertical and the horizontal rook movement must be $90° - [\alpha, \beta, \theta]$ (Laisin, Okeke & Chukwuma, 2020).

Theorem. II.1.2

The number of ways of placing t non-attacking rooks on the full $m \times n$ board is equal to

$(mt)(nt)t!$, (Nicholas, and Feryal, 2009)

Theorem. II.1.3

Let A and B be boards that share no rows or columns. The rook polynomial for the board

$A \cup B$ consisting of the union of the tiles in A and B is;

$R_{A \cup B}(x) = R_A(x) \times R_B(x)$, (Laisin, 2017; Nicholas, and Feryal, 2009)

Theorem. II.1.4

Let A be an $m \times n$ board with restrictions, A^c be the complement of A, and

$$R_{A^c}(x) = r_0 + r_1 x + r_2 x^2 + \ldots + r_t x^t \ldots$$
be the rook polynomial for A^c. Then the number of ways to place t non-attacking rooks on A is
$$\sum_{i=0}^{t} (-1)^i (m - it - i)(n - it - i)(t - i)!\, r!,$$
(Nicholas, and Feryal, 2009)

Theorem. II.1.5
The number of ways to place t non-attacking rooks on a triangle board of size m is equal to $S(m+1, m+1-k)$, where $1 \le k \le m$ and $S(n,k)$ are the Stirling numbers of the second kind.

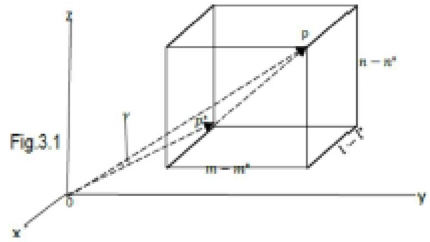
III. MAIN RESULTS

Theorem III.1 Angle between two vectors

Suppose γ is the angle between two vectors. Then, the sum of products of the corresponding direction of the rook movement is the cosines from the two generated vectors by the restricted area. Hence, the rook movement is a three-dimensional structure.

Proof

Let $\underline{a}$ be one vector with direction cosines [l, m, n] and b be the other vector with direction cosines $[l^* m^* n^*]$. Task: To find the angle between these two vectors let OP and OP^* be unit vectors parallel to a and b respectively. Then p has coordinates [l, m, n] and p^* has coordinates $[l^* m^* n^*]$.



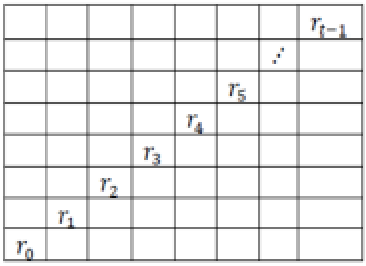
Now, considering fig 3.1 we have that; $(pp^*)^2 = (l - l^*)^2 + (m - m^*)^2 + (n - n^*)^2$
$$= (l^2 + m^2 + n^2) + [(l^*)^2 + (m^*)^2 + (n^*)^2] - 2(ll^* + mm^* + nn^*)$$
but we have $l^2 + m^2 + n^2 = 1$ and also $(l^*)^2 + (m^*)^2 + (n^*)^2 = 1$ then,
$$(pp^*)^2 = 2 - 2(ll^* + mm^* + nn^*) \qquad (3.1)$$
for vector (3.1)also, by cosine rule we have
$$(pp^*)^2 = op^* + op^{*2} - (op)(op^*)cos\gamma$$
$$= 2 - 2cos\gamma \qquad (3.2)$$
Now, from (3.1) and (3.2) we have $(pp^*)^2 = 2 - 2(ll^* + mm^* + nn^*) = 2 - 2cos\gamma$
$$\Rightarrow cos\gamma = ll^* + mm^* + nn^*$$
Thus, the sum of the rook movement is the product of the corresponding direction of the rook movement as its cosines of the given vector that give the three- dimensional structure.

THEOREM III.2

Suppose B is a 3-demensional disjoined rook boards with forbidden squares and the t non-attacking rook movements generate a rook function, then, the generating rook function is;
$$R(x, B_1, B_2) = \sum_{i=0}^{t-1} \prod_{s=0}^{t-1} X_{Bj,S}(x)^i r_i(B_j)[1 + \omega sign(v_1 \times v_2 \cdot v_3)], \forall j = 1, 2, \cdots, t-1$$
where ω is the number of sides for the higher dimensional space for the given board.

Proof

Considering, B_1 we have for k non attacking rooks on a disjoined chess board as follows;
$$R(x, B_1) = \sum_{i=0}^{t-1} (x)^i r_i(B_1)$$
$$= 1 + x r_1(B_1) + x^2 r_2(B_1) + \ldots + x^{t-1} r_{t-1}(B_1)$$
Since, B_1 is a two-dimensional chess board, then, the total number of rooks that give a maximum number of rook movements on the forbidden space with the t non-attacking rooks is as follows;



B_1

We have the following rook placements; $r_0(x, B_1) = 1$, $r_1(x, B_1)$, $r_2(x, B_1)$, $r_3(x, B_1)$, $r_4(x, B_1)$, $r_5(x, B_1)$, $r_6(x, B_1), \cdots$, $r_{t-1}(x, B_1)$ respectively.
However, the two-dimensional rook boards generate the rook function with the generating function.
$$R(x, B_1) = \sum_{i=0}^{t-1} (r_0(x, B_1), \; r_1(x, B_1), \; r_2(x, B_1), \; \cdots, \; r_{t-1}(x, B_1))x^i$$
$$= r_0(x, B_1) + r_1(x, B_1)x + r_2(x, B_1)x^2 + \cdots + r_{t-1}(x, B_1)x^{t-1}$$
Thus, $R(x, B_1)$ is the pathway generated by the rook movements for the horizontally and vertically rook movements. Hence, a projection of the vector of $(ti + tj + tk$ on the vector $i + j + k$ has a unit vector that is given as $\frac{1}{\sqrt{3}}(i + j + k)$ that gives us a rook projection movement of $\frac{3t}{\sqrt{3}}$ $units$. As required.

Now, the three-dimensional rook movement is generated is denoted as $R(x, B_1, B_2)$. Since the chess boards decompose into disjoint boards with vectors v_1, v_2 and v_3 respectively.

then, it follows the placement of rooks on the chess board, thus we have;
$$R(x, B_1, B_2) = R(x, B_1)x^i + R(x, B_2)\omega sign(v_1 \times v_2 \cdot v_3)x^i, \forall j = 1, 2, \cdots, t-1$$
$$= [[r_0(B_1) + r_1(B_1) + \cdots + r_{t-1}(B_1)]x^i + \omega[r_0(B_2) + r_1(B_2) + r_2(B_2) + \cdots + r_{t-1}(B_2)]]x^i$$

$$= \sum_{i=0}^{t-1} r_i(x, B_1, B_2)$$

$$= \sum_{i=0}^{t-1} \prod_{S=0}^{t-1} X_{B_j,S}(x)^i r_i(B_j) + \sum_{i=0}^{t-1} \prod_{S=0}^{t-1} X_{B_j,S}(x)^i r_i(B_{2j})\omega sign(v_1 \times v_2 \cdot v_3)$$

$$= \sum_{i=0}^{t-1} \prod_{s=0}^{t-1} X_{B_j,S}(x)^i r_i(B_j)[1 + \omega sign(v_1 \times v_2 \cdot v_3)], \ \forall j = 1,2,\cdots,t-1;$$

where ω is the number of sides for the higher dimensional space for the given board

■

IV. NUMERICAL APPLICATIONS

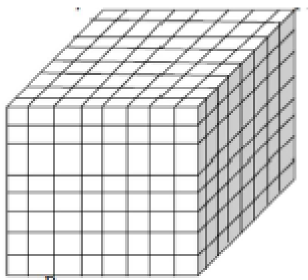
Example IV.1

Determine the generating function of an 8×8 3-dimensional chess board with non-attacking rooks on the forbidden space. Hence, find its rook projection on the vector $i + j + k$.

Solution

Recall that;

$R(x, B_1, B_2) = R(x, B_1)x^i + 5R(x, B_2)sign(v_1 \times v_2 \cdot$ *v3xi, ∀i=1,2,⋯, 7=i=07s=07XBj,SxiriBj1+ 5sign(v1 × v2 · v3), ∀j=1,2,⋯, 7*

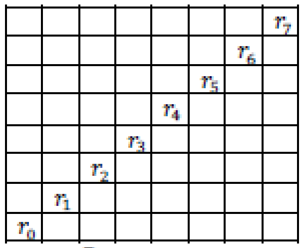


B_2

Now considering, B_1 we have for 8 non attacking rooks on a disjoined chess board as follows;

$$R(x, B_1) = \sum_{i=0}^{7} (x)^i r_i(B_1) = 1 + xr_1(B_1) + x^2 r_2(B_1) + \ldots + x^7 r_7(B_1)$$

Since, B_1 is a two-dimensional chess board, then, the total number of rooks that give a maximum number of rooks moves on the forbidden space with non-attacking rooks is as follows;



B_1

We have the following rook placements; $r_0(x, B_1) = 1$, $r_1(x, B_1) = 8 ways$, $r_2(x, B_1) = 21\ ways$, $r_3(x, B_1) = 20\ ways$, $r_4(x, B_1) = 5\ ways$, $r_5(x, B_1) = 4\ ways$, $r_6(x, B_1) = 2\ ways\ and\ r_7(x, B_1) = 1\ way$ respectively.

However, the two-dimensional rook boards generate the rook function with the generating function.

$$R(x, B_1) = \sum_{i=0}^{7} (1, 8, 21, 20, 5, 4, 2, 1)x^i = 1 + 8x + 21x^2 + 20x^3 + 5x^4 + 4x^5 + 2x^6 + x^7$$

Thus, $R(x, B_1)$ is the pathway generated by the rook movements for the horizontally and vertically moves.

Now, for the 3-dimensional structure, we have;

$$R(x, B_1, B_2) = R(x, B_1)x^i + 5R(x, B_2)sign(v_1 \times v_2 \cdot v_3)x^i = \sum_{i=0}^{8-1} (1, 8, 21, 20, 5, 4, 2, 1)x^i[1 + 5sign(v_1 \times v_2 \cdot v_3)], \ \forall j = 1,2,\cdots,7$$

$$= 6 + 48x + 126x^2 + 120x^3 + 40x^4 + 32x^5 + 16x^6 + 6x^7$$

Thus, $R(x, B_1, B_2)$ is the pathway generated by the rook movements for the 3-dimensional structure, when the 8×8 is a forbidden area.

Hence, a projection of the vector of $8i + 8j + 8k$ on the vector $i + j + k$ has a unit vector that is given as $\frac{1}{\sqrt{3}}(i + j + k$ that gives us a rook projection movement of *243 units*. As required.

V. CONCLUSION

The research was able to bring out the history for the chess game, construct and show that, the sum of the rook movement is the product of its corresponding direction of the angle between the two vectors that give the three-dimensional structure. However, we were also able to generate a rook function for a three-dimensional rook boards with forbidden squares that is generated by the vector's directives.

VI. RECOMMENDATIONS

Furthermore, studies can be carried out with the bishops on a three-dimensional structure to determine a general pathway for bishop movements on a three- dimensional chess board. In addition, further studies could also examine what would happen to bishop/rook polynomials by changing the shape of the boards.

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