

Efficiency of Two Stage Stratified Sampling Procedure under Gamma Distribution

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Abstract— In this study, simple random sampling was used to select 4 primary units out of 8 primary units and 8 secondary units out of 20 secondary units using Fisher and Yates random number table. The population for this research study is sets of data simulated with R-package of a gamma distribution with different scales and each scale contains different shapes. The scales are used as the stratification variable for primary unit while the shape serves as the stratification variable for the secondary unit or sub-unit, samples from the data simulated were then taken and analyze for precisions.

Index Terms— Fisher and Yates, Simulated, gamma distribution, precision, R-package, stratification, precision

I. INTRODUCTION

Sampling selection methods in research is very important. Irrespective of the study area (sciences, psychological, social sciences etc), the main objective is to be able to make valid and generalized output that represent the whole study area. Research limitation which time and resources are some among many problems associated with research, demand the need for samples to be selected from the entire population to save cost and timing. In general, the method of sampling is divided into two viz;

- (i) Probability/Random sampling technique
- (ii) Non-Probability/Non-Random sampling technique

Stratified sampling falls under the probability/random method of sampling; hence, the name stratified random sampling emerges. For stratified sampling, the population is divided into groups called strata after which a simple random method is used in selecting samples from the strata. Precision is determined using the variances from the simulated gamma distribution population using R- package.

A. POPULATION OF STUDY

The population under study in this research are set of data simulated with R-Package which follows gamma distribution with different scales and each scale contains different shapes i.e. the scale serves as the stratification variable for the primary unit while the shape serves as the stratification variable for the secondary unit or sub unit, we decided to take samples from the data simulated. The selected samples are used to conduct analysis in the next chapter.

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B. METHOD OF SAMPLING

For the purpose of this study, simple random sampling has been used to pick 4 primary units out of 8 primary units and 8 secondary units out of 20 secondary units using simple random sampling. The simple random sampling is done with Fisher and Yates random number table.

C. SAMPLE SIZE

As it has been discussed above that sample selected for analysis of this study is 4 primary units and 8 secondary units.

II. METHODOLOGY

A. The Parameters/Estimators used

Overall population mean per sub unit

$$\bar{Y} = \frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M Y_{ij} \dots \dots \dots (i)$$

Sample mean per sub unit

$$\bar{y} = \frac{1}{nm} \sum_{i=1}^n \sum_{j=1}^m y_{ij} \dots \dots \dots (ii)$$

Variance among primary unit means

$$S_b^2 = \frac{1}{N-1} \sum_{i=1}^N (\bar{Y}_i - \bar{Y})^2 \dots \dots \dots (iii)$$

Variance among subunit within primary unit

$$S_w^2 = \frac{1}{N(M-1)} \sum_{i=1}^N \sum_{j=1}^M (Y_{ij} - \bar{Y}_i)^2 \dots \dots \dots (iv)$$

Variance among sampled primary unit means

$$s_b^2 = \frac{1}{n-1} \sum_{i=1}^n (\bar{y}_i - \bar{y})^2 \dots \dots \dots (v)$$

Variance among sampled subunits within primary unit

$$s_w^2 = \frac{1}{n(m-1)} \sum_{i=1}^n \sum_{j=1}^m (y_{ij} - \bar{y}_i)^2 \dots \dots \dots (vi)$$

Variance of the sample mean



$$V(\bar{y}) = \left(\frac{N-n}{Nn}\right) s_b^2 + \left(\frac{M-m}{Mm}\right) \frac{s_w^2}{n} \dots \dots \dots \text{(vii)}$$

For its precision

$$P(\bar{y}) = \frac{S.E(\bar{y})}{\bar{y}} \times 100\% \dots \dots \dots \text{(viii)}$$

For the population total

$$\hat{Y} = NM\bar{y} \dots \dots \dots \text{(ix)}$$

And its variance

$$V(\hat{Y}) = N^2M^2V(\bar{y}) \dots \dots \dots \text{(x)}$$

$$V(\hat{Y}) = NM^2 \left(\left(\frac{N-n}{n}\right) s_b^2 + N^2M \left(\frac{M-m}{m}\right) \frac{s_w^2}{n} \right) \dots \dots \dots \text{(xi)}$$

For its precision

$$P(\hat{Y}) = \frac{S.E(\hat{Y})}{\hat{Y}} \times 100\% \dots \dots \dots \text{(xii)}$$

FOR TWO STAGE STRATIFIED SAMPLING

For sample mean

$$\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi} \dots \dots \dots \text{(xiii)}$$

$$\text{Where } \bar{x}_h = \frac{1}{m_h} \sum_{i=1}^{m_h} x_{hi} \dots \dots \dots \text{(xiv)}$$

And its variance

$$V(\bar{x}_h) = \left(\frac{1-f_{h1}}{n_h}\right) S_{h1}^2 + f_{h1} \left(\frac{1-f_{h2}}{m_h n_h}\right) S_{h2}^2 \dots \dots \dots \text{(xv)}$$

$$\text{Where } f_{h1} = \frac{n_h}{N_h} \text{ \& } f_{h2} = \frac{m_h}{M_h}$$

Variance between primary units mean

$$S_{h1}^2 = \sum_{h=1}^{n_h} \frac{(\bar{x}_h - \bar{\bar{x}}_h)^2}{n_h - 1} \dots \dots \dots \text{(xvi)}$$

Variance among subunits within primary unit

$$S_{h2}^2 = \sum_{h=1}^{n_h} \sum_{i=1}^{m_h} \left(\frac{(x_{hi} - \bar{x}_h)^2}{n_h(m_h - 1)} \right) \dots \dots \dots \text{(xvii)}$$

For the population

$$\bar{X}_{st} = \sum_{h=1}^L \left\{ \frac{N_h M_h \bar{x}_h}{\sum_{h=1}^L N_h M_h} \right\} \dots \dots \dots \text{(xviii)}$$

$$\bar{X}_{st} = \sum_{h=1}^L W_h \bar{x}_h$$

And its variance

$$V(\bar{X}_{st}) = \sum_{h=1}^L \left\{ W_h^2 \left(\frac{1-f_{h1}}{n_h} \right) S_{h1}^2 + \left(\frac{f_{h1}(1-f_{h2})}{n_h m_h} \right) S_{h2}^2 \right\} \dots \dots \dots \text{(xix)}$$

For its precision

For the population total

$$P(\bar{X}_{st}) = \frac{S.E(\bar{X}_{st})}{\bar{X}_{st}} \times 100\% \dots \dots \dots \text{(xx)} \quad \hat{X}_{st} = N_h M_h \bar{x}_h \dots \dots \dots \text{(xxi)}$$

And its variance,

$$V(\hat{X}_{st}) = \left[\sum_{h=1}^L \left(\frac{N_h M_h}{\sum_{h=1}^L N_h M_h} \right)^2 \right] V(\bar{X}_{st}) \dots \dots \dots \text{(xxii)} \quad \square V(\hat{X}_{st}) = \frac{N_h^2(1-f_{h1})}{n_h} S_{h1}^2 + \frac{N_h}{n_h} \sum M_{hi}^2 \left(\frac{1-f_{h2}}{m_{hi}} \right) S_{h2}^2$$

For its precision

$$P(\hat{X}_{st}) = \frac{S.E(\hat{X}_{st})}{\hat{X}_{st}} \times 100\% \dots \dots \dots \text{(xxiii)}$$

Table 1.1: SOME USEFUL NOTATIONS IN THE ANALYSIS

Notation	Meaning
N_h	Number of primary units in stratum h
M_h	Number of elements in each of the primary unit in stratum h
n_h	Number of primary unit selected in stratum h
m_h	Number of element selected from each selected sampling unit in stratum h
\bar{x}_h	Sample mean for subunit in the stratum h
$\bar{\bar{x}}_h$	Overall sample mean per stratum

S_{h1}^2	Variance between primary unit means in stratum h
S_{h2}^2	Variance within primary unit means in stratum h
$V(\bar{X}_h)$	Variance of the overall sample mean per stratum
\bar{X}_{st}	Estimate of population mean
\hat{X}_{st}	Estimate of population total
$V(\bar{X}_{st})$	Estimate of variance of the population mean
$V(\hat{X}_{st})$	Estimate of the variance of the population total

ESTIMATION OF POPULATION PARAMETERS IN TWO STAGE STRATIFIED SAMPLING

Table 1.2: Primary unit samples ($n_h = 4$), Secondary unit samples ($m_h=4$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A	2.9663	2.331	1.9591	0.3888	Lower	Upper
B	2.3519	1.1945	0.2637	0.1845	4.04	5.87
C	6.4481	8.8998	3.7161	1.4093	6466.18	9397.82
D	8.0638	9.7125	2.8109	1.5131	Precision	9.43%
\bar{X}_{st}	4.9575	$V(\bar{X}_{st})$	0.2185			
\hat{X}_{st}	7932	$V(\hat{X}_{st})$	559307			

Table 1.3: Primary unit samples ($n_h = 4$), Secondary unit samples ($m_h=8$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A	2.8275	1.858757	1.177258	0.287643	Lower	Upper
B	2.214375	1.20068	0.353696	0.182755	4.18	5.89
C	6.7225	8.791434	3.173478	1.342516	6694.44	9420.56
D	8.3794	7.875496	3.798617	1.209814	Precision	8.63%
\bar{X}_{st}	5.0360	$V(\bar{X}_{st})$	0.1889			
\hat{X}_{st}	8057.50	$V(\hat{X}_{st})$	483636.4829			

Table 1.4: Primary unit samples ($n_h = 4$), Secondary unit samples ($m_h=12$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A	2.814	1.9419	0.9577	0.2945	Lower	Upper
B	2.165	1.3359	0.4492	0.2019	4.25	5.96
C	6.6481	7.5011	3.5789	1.1371	6794.16	9530.51
D	8.7788	9.2875	5.6558	1.412	Precision	8.55%
\bar{X}_{st}	5.1015	$V(\bar{X}_{st})$	0.19034			
\hat{X}_{st}	8162.33	$V(\hat{X}_{st})$	487269.5			

Table 1.5: Primary unit samples ($n_h = 4$), Secondary unit samples ($m_h=16$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A	2.7748	1.9292	0.9631	0.2906	Lower	Upper
B	2.2466	1.2630	0.5187	0.1900	4.30	6.04
C	6.7831	7.0171	3.8964	1.0574	6884.45	9658.30
D	8.8739	10.5553	6.4820	1.5914	Precision	8.56%
\bar{X}_{st}	5.1696	$V(\bar{X}_{st})$	0.1956			
\hat{X}_{st}	8271.38	$V(\hat{X}_{st})$	500720.8			

Table 1.6: Primary unit samples ($n_h=5$), Secondary unit samples ($m_h=4$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A		1.83506	1.7576		Lower	Upper
	2.8345	7	15	0.218659		
B		0.95515	0.2705			
	2.243	1	87	0.100927	4.15	5.49
C		6.69411	3.1480			8791.
	6.386	4	83	0.732373	6647.62	18
D		7.54602	3.1151		Precision	7.08
	7.835	5	93	0.816906		%
\bar{X}_{st}	4.8246	$V(\bar{X}_{st})$	0.116804			
\hat{X}_{st}	7719.4	$V(\hat{X}_{st})$	299018.5			

Table 1.7: Primary unit samples ($n_h=5$), Secondary unit samples ($m_h=8$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A		1.4091	1.0561		Lower	Upper
	2.7728			0.1488		r
B		0.9249	0.3189		4.23	5.48
C		7.1733	3.0122			8773.
	6.3820			0.7399	6769.50	30
D		6.2194	3.6733		Precision	6.58
	8.1293			0.6495		%
\bar{X}_{st}	4.8571	$V(\bar{X}_{st})$	0.1021			
\hat{X}_{st}	7771.4	$V(\hat{X}_{st})$	261299.6			

Table 1.8: Primary unit samples ($n_h=5$), Secondary unit samples ($m_h=12$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A		1.58534	0.96361		Lower	Upper
	2.823833	5	6	0.161747		r
B		0.9431	0.3646		4.31	5.57
C		5.8012	3.8322			8912.
	6.4608			0.5929	6895.69	84
D		7.87834			Precision	6.51%
	8.3515	7	5.0842	0.8048		
\bar{X}_{st}	4.9402	$V(\bar{X}_{st})$	0.1034			
\hat{X}_{st}	7904.27	$V(\hat{X}_{st})$	264792.50			

Table 1.9: Primary unit samples ($n_h=5$), Secondary unit samples ($m_h=16$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A		1.5250	1.0099		Lower	Upper
	2.8020			0.1537		er
B		0.8924	0.4354		4.40	5.66
C		5.4724	4.0199			9056.
	6.5784			0.5523	7035.95	45
D		8.5723	6.0210		Precisio	6.41
	8.5118			0.864751	n	%
\bar{X}_{st}	5.0289	$V(\bar{X}_{st})$	0.1038			
\hat{X}_{st}	8046.2	$V(\hat{X}_{st})$	265674.29			

Table 2.1: Primary unit samples ($n_h=6$), Secondary unit samples ($m_h=4$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A	2.7958	1.4770	1.6759	0.1320	Lower	Upper
B	2.1192	0.8561	0.2292	0.0617	4.09	5.15
C	6.2371	5.4883	2.7175	0.4202	6549.19	81
D	7.3417	7.4971	2.8542	0.5569	Precisio	5.85
\bar{X}_{st}	4.6234	$V(\bar{X}_{st})$	0.0732		n	%
\hat{X}_{st}	7397.5	$V(\hat{X}_{st})$	187324.08			

Table 2.2: Primary unit samples ($n_h=6$), Secondary unit samples ($m_h=8$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A	2.6904	1.16791	1.0435	0.0857	Lower	Upper
B	2.0717	0.77176	0.2993	0.0537	4.18	5.16
C	6.2077	5.92088	2.6538	0.4146	6694.24	43
D	7.7110	6.02493	3.9086	0.4310	Precisio	5.31
\bar{X}_{st}	4.6702	$V(\bar{X}_{st})$	0.0616		n	%
\hat{X}_{st}	7472.33	$V(\hat{X}_{st})$	157598.2			

Table 2.3: Primary unit samples ($n_h=6$), Secondary unit samples ($m_h=12$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A	2.6911	1.3740	0.9494	0.0948	Lower	Upper
B	2.1217	0.7545	0.4588	0.0518	4.27	5.24
C	6.2292	4.9630	3.3613	0.3421	6826.51	8381.8
D	7.9685	7.1829	4.9065	0.4952	Precisio	5.22%
\bar{X}_{st}	4.7526	$V(\bar{X}_{st})$	0.0615		n	
\hat{X}_{st}	7604.17	$V(\hat{X}_{st})$	157421.40			

Table 2.4: Primary unit samples ($n_h=6$), Secondary unit samples ($m_h=16$)

STRATUM	\bar{x}_h	S_{h1}^2	S_{h2}^2	$V(\bar{x}_h)$	C. I.	
A	2.7440	1.2394	1.1627	0.0841	Lower	Upper
B	2.1843	0.7231	0.4959	0.0488	4.33	5.31
C	6.3116	4.8051	3.5518	0.3248	6926.48	8501.5
D	8.0452	8.1638	5.6430	0.5513	Precisio	5.21%
\bar{X}_{st}	4.8213	$V(\bar{X}_{st})$	0.0631		n	
\hat{X}_{st}	7714	$V(\hat{X}_{st})$	161438.30			

Interpretations:

From table 1.2 above, it can be deduced that the two-stage

stratified mean and total of the population characteristics which follows gamma distribution is 4.9575 and 7932.

Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.04 and 6466.18 respectively and at the same time, it can never

From table 1.3 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 5.0360 and 8057.50. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.18 and 6694.44 respectively and at the same time, it can never exceed above 5.89 and 9420.56 i.e. the 95% confidence interval is $4.18 < \bar{X}_{st} < 5.89$ and $6694.44 < \hat{X}_{st} < 9420.56$.

From table 1.4 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 5.1015 and 8162.33. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.25 and 6794.16 respectively and at the same time, it can never exceed above 5.96 and 9530.51 i.e. the 95% confidence interval is $4.25 < \bar{X}_{st} < 5.96$ and $6794.16 < \hat{X}_{st} < 9530.51$.

From table 1.5 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 5.1696 and 8271.38. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.30 and 6884.45 respectively and at the same time, it can never exceed above 6.04 and 9658.30 i.e. the 95% confidence interval is $4.30 < \bar{X}_{st} < 6.04$ and $6884.45 < \hat{X}_{st} < 9658.30$.

From table 1.6 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 4.8246 and 7719.4. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.15 and 6647.62 respectively and at the same time, it can never exceed above 5.49 and 8791.18 i.e. the 95% confidence interval is $4.15 < \bar{X}_{st} < 5.49$ and $6647.62 < \hat{X}_{st} < 8791.18$.

From table 1.7 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 4.8571 and 7771.4. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.23 and 6769.50 respectively and at the same time, it can never exceed above 5.48 and 8773.30 i.e. the 95% confidence interval is $4.23 < \bar{X}_{st} < 5.48$ and $6769.50 < \hat{X}_{st} < 8773.30$.

From table 1.8 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 4.9402 and 7904.27. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.3098 and 6895.69 respectively and at the same time, it can never exceed above 5.48 and 8773.30 i.e. the 95% confidence interval is $4.31 < \bar{X}_{st} < 5.57$ and $6895.69 < \hat{X}_{st} < 8912.84$.

From table 1.9 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 5.0289 and 8046.2. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.40 and 7035.95 respectively and at the same time, it can never exceed above 5.66 and 9056.45 i.e. the 95% confidence

exceed above 5.87 and 9397.82 i.e. the 95% confidence interval is $4.04 < \bar{X}_{st} < 5.87$ and $6466.18 < \hat{X}_{st} < 9397.82$.

interval is $4.40 < \bar{X}_{st} < 5.66$ and $7035.95 < \hat{X}_{st} < 9056.45$.

From table 2.1 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 4.6234 and 7397.5. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.09 and 6549.19 respectively and at the same time, it can never exceed above 5.15 and 8245.81 i.e. the 95% confidence interval is $4.09 < \bar{X}_{st} < 5.15$ and $6549.19 < \hat{X}_{st} < 8245.81$.

From table 2.2 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 4.6702 and 7472.33. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.18 and 6694.24 respectively and at the same time, it can never exceed above 5.16 and 8250.43 i.e. the 95% confidence interval is $4.18 < \bar{X}_{st} < 5.16$ and $6694.24 < \hat{X}_{st} < 8250.43$.

From table 2.3 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 4.7526 and 7604.17. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.27 and 6826.51 respectively and at the same time, it can never exceed above 5.24 and 8381.82 i.e. the 95% confidence interval is $4.27 < \bar{X}_{st} < 5.24$ and $6826.51 < \hat{X}_{st} < 8381.82$.

From table 2.4 above, it can be deduced that the two-stage stratified mean and total of the population characteristics which follows gamma distribution is 4.8213 and 7714. Statistically, we are 95% sure that if the total population is being investigated, the outcome can never fall below 4.33 and 6926.48 respectively and at the same time, it can never exceed above 5.31 and 8501.52 i.e. the 95% confidence interval is $4.33 < \bar{X}_{st} < 5.31$ and $6926.48 < \hat{X}_{st} < 8501.52$.

III. CONCLUSION

From all these analyses, despite the fact that all the estimates of the two stage stratified mean are all reliable as all the precisions are less than 10%, meanwhile, the sample of 6 primary units with 16 secondary unit yielded the least precision with 5.21% among all other samples selected, therefore we can conclude that the sample of 6 primary units with 16 secondary units selected provided the most efficient estimator of the population characteristics which follows gamma distribution.

IV. RECOMMENDATION

Based on the analysis and conclusion so far, it can be recommended that to get most efficient estimator from a large scale sample survey covering an entire country or region, especially when national estimates as well as estimate for each geo-graphical or political region making up the country are required, an enumerator should make use of two stage stratification approach, also during the process he should select 50% of the primary units.

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