Unification of Fundamental Forces

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***Abstract*— In this research, we used both statistical and analytical methods to obtain relations that may suggestively indicate unification of the four fundamental interactions. These interactions include gravitational, electromagnetic, weak (nuclear) and strong (nuclear) forces.**

***Index Terms*— Blackhole masses, GUT Scale, Dirac Equations, Klein-Gordon Equations.**

I. Introduction

A unified theory of interaction, as it is generally understood implies a description of the four fundamental forces such as gravitation (between particles with mass), electromagnetic (between particles with charge/magnetism), strong interaction (between quarks) and weak interaction (operates between neutrinos and electrons). Based on the foundations of the Maxwell's equations, Physicists are trying to combine the four fundamental forces in a single mathematical formulation. In the classical solutions of long ranged interactions, work is done in some other studies related to a unified Equation (Lalit et al., 2011).

II. MATERIALS AND METHOD

We employed both statistical and analytical method to obtain relations that may suggestively indicate unification of the four fundamental interactions.

Statistical Data Analysis

Here, we first tried to use statistical data analysis to obtain a relation that suggestively may unify the electromagnetic and gravitational forces (Electro-gravity). The data used in the analyses were obtained from Minfeng et al. (2001), Sofia et al. (1999), Jong-hak and Megan (2002) and Yi Liu et al. (2005).

The magnetic field energy density U_{mag}, of a source emitting synchrotron radiation is given by Robson (1996).

$$\frac{dE}{dt} = \frac{4}{3}\sigma_T c\gamma^2 U_{mag} \quad (1)$$

where E is energy, t is time, σ_T is Thompson cross-section, c is speed of light, γ is Lorentz factor. By definition, $\frac{dE}{dt}$ is luminosity. Therefore, assuming bolometric luminosity P_{bol} of the source, and taking σ_T to be $6.65 \times 10^{-29} m^2$ (Robson 1996). Equation (1) may be written as:

$$U_{mag} = \frac{3P_{bol}}{4\sigma_T c\gamma^2} \quad (2)$$

The last equation simply indicates that we can estimate the value of the source magnetic field energy density once its bolometric luminosity and jet speed are known. Moreover, gravitational force, F_g of interaction of a given blackhole is given by:

$$F_g = 2 \times 10^{-6} \frac{\pi k}{\hbar} \frac{m_\odot}{m_b} \quad \text{(Hawking, 1976)} \quad (3)$$

where k is Boltzmann constant, $m_\odot$ is solar mass, m_b is mass of blackhole. Using equation (3), we also estimated gravitational force of interaction of each source. Scatter plot of F_g against U_{mag} was carried out and is shown in figure 1.

Analytical Method

We used analytical method to obtain a mathematical model which suggestively may unify the four fundamental interactions. It was suggested that in the unified theory of interactions, Klein Gordon equation and Dirac equation are the two powerful candidates for unification. The Dirac equation includes the electromagnetic interaction (Wagener, 2009). Klein-Gordon and Dirac equations are analogs of the Schrodinger equation which tries to make quantum mechanics compatible with special relativity, unlike the Schrodinger equation which is compatible only with Galilean relativity (Slawianowski and Kovalchuk, 2002).

III. RESULTS

With a good correlation coefficient ($R \cong 0.6$) from the plot in figure 1 below, we obtained an equation given by:

$$\log F_g = -0.740 \log U_{mag} + 45.46 \quad (4)$$

Simplifying, we obtain:

$$F_g = (2.884 \times 10^{45}) U_{mag}^{-0.74} \quad (5)$$

The last equation simply suggested that F_g varies with U_{mag} according to the relation:

$$F_g \sim U_{mag}^{-\psi} \quad (6)$$

where $\psi = 0.74$ is the slope of the plot.

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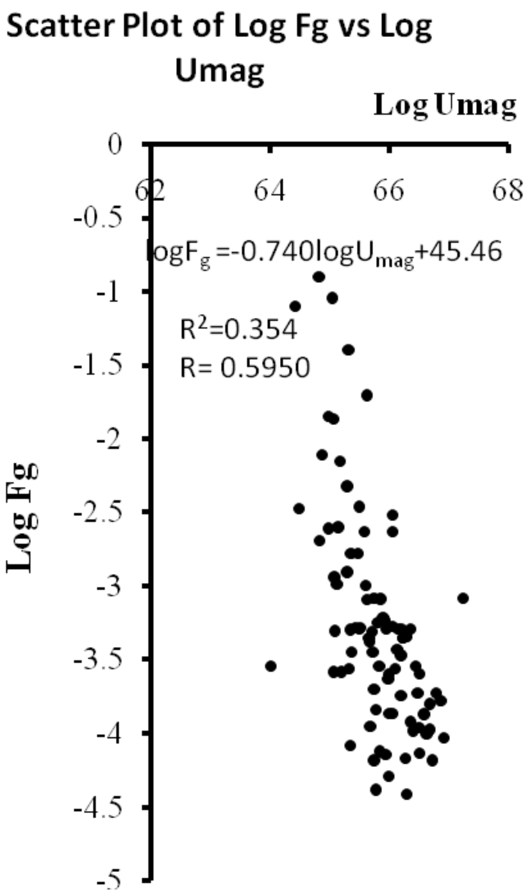

Figure 1: The plot of unification of gravitational and electromagnetic interaction

For the analytical method, our attempt was first, to derive the Dirac equation. Therefore, we started by using the equation of energy and momentum operators (Rosen, 1994), which are shown below respectively.

$$\hat{E} = i\hbar\frac{\partial}{\partial t} \qquad (7)$$

$$\hat{p} = -i\hbar\nabla \qquad (8)$$

i.e. $\hat{p} = -i\hbar(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})$; where $\hbar$ is the Dirac constant measured in J.s (i.e $\frac{h}{2\pi}$).

By definition, $H = T + V$ or $\hat{H} = \hat{T} + \hat{V}$ (i.e. in terms of operator) (9)

where $\hat{T} = \frac{\hat{p}.\hat{p}}{2m} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$ (10)

However, in a zero potential field, $V(r) = 0$; hence equation (9) and (10) yield:

$$\hat{H} = \frac{\hat{p}^2}{2m} \qquad (11)$$

Recalling $\hat{E}^2 = c^2\hat{p}^2 + (mc^2)^2$ (Huegele *et al.*, 2014) (12)

From relativistic quantum mechanics, we noted that: $\hat{E}^2\psi = \hat{H}^2\psi$ (13)

where E is a real number called the energy of state. Equation (13) is the eigenvalue equation of the operator H; the application of H on the eigenfunction ψ yields the same function, multiplied by the corresponding eigenvalue E. The allowed energies are therefore the eigenvalues of the operator H.

Putting equation (8) into equation (12), we have:

$$\hat{H}^2 = c^2(-i\hbar\nabla)^2 + (mc^2)^2 \qquad (14)$$

The new Schrödinger equation for this becomes:

$$\hat{H}^2\psi = [c^2(-i\hbar\nabla)^2 + (mc^2)^2]\psi = \hat{E}^2\psi \qquad (15)$$

by multiplying through with the wave function ψ where $\psi(x,t) = Aexpi(kx - \omega t)$ (16)

Hence, equation (15) is called the Klein-Gordon equation.

Pollock (2010) suggested that: $H_{Dir} = c\vec{\alpha}.\vec{p} + \beta mc^2$ (17)

The square of this has to be the same as the Klein-Gordon Hamiltonian (Mrato 1991):

$$H_{Dir}^2 = c^2p^2 + (m_0c^2)^2 \qquad (18)$$

where, E, p, m_0 are the energy, momentum, and rest mass energy respectively and α_i and β are matrices.

A set of vectors $\{\alpha_i\}$ is orthogonal if any pair of two separate elements is orthogonal, that is, $\langle\alpha_i, \beta\rangle = 0$ for $i \neq j$. In particular, the set is orthonormal if in addition each of its elements is a unit vector.

Thus, $H_{Dir}^2 = c^2(\vec{\alpha}.\vec{p})^2 + \beta^2(mc^2)^2 = c^2p^2 + (m_0c^2)^2$ (19)

Therefore,

$$(\vec{\alpha}.\vec{p})^2 = \sum_{ij}\alpha_i\alpha_j p_i p_j = \frac{1}{2}\sum_{ij}[\alpha_i\alpha_j + \alpha_j\alpha_i]\, p_i p_j = p^2 \qquad (20)$$

This is possible only if $\frac{1}{2}[\alpha_i\alpha_j + \alpha_j\alpha_i] = \delta_{ij}$ (21)

where α_i and β cannot be ordinary numbers but matrices and δ_{ij} is the kronecker delta function.

To obtain the final Dirac equation, we use the energy relation of a spin -1/2, charge particle (q = -e), which can bound in an electromagnetic potential, $A_\mu(r,t)$; and can be written in linear form as:

$$\gamma_0(\hat{E} - eA_0) - \gamma.\left(p - \frac{e}{c}\vec{A}\right)c - mc^2 = 0 \qquad (22)$$

where γ_0 and γ are matrices, and using the anticommutation properties of the γ matrices, we get the relativistic energy-momentum relation:

$$\hat{E} = \left(p - \frac{e}{c}\vec{A}\right)c - eA_0 + mc^2 + \sqrt{2EeA_0} \qquad (23)$$

The equation (23) is then multiplied by the wave function ψ to get:

$$\hat{E}\psi = \left(\left(p - \frac{e}{c}\vec{A}\right)c - e\phi + mc^2 + \sqrt{2EeA_0}\right)\psi \qquad (24)$$

noting that $A_0 = \phi$.

Relating $H_{Dir} = c\vec{\alpha}.\vec{p} + \beta mc^2$; $\vec{p} = p - \frac{e}{c}\vec{A}$ and $\hat{E}\psi = i\hbar\frac{\partial\psi}{\partial t}$ (Comay, 2005) with equation (24), we obtained:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(c\vec{\alpha}\left(p - \frac{e}{c}\vec{A}\right) - e\phi + \beta mc^2 + \sqrt{2EeA_0}\right)\psi$$

(25)

where $c\vec{p} = [mc^2] = [e\phi] = energy$, ϕ is the electrostatic potential, A is the vector potential, c is the speed of light, p is the momentum, $\hbar$ is Dirac constant and σ_i *are the* 2×2 pauli matrices.

Our Newtonian-type model of gravitation is based on a lagrangian:

$$L = -m_0(c^2 - v^2)e^{R/r} \quad (26)$$

where m_0 is gravitational mass of a particle, c is speed of light, R is the Schwarzschild radius which is equal to $\frac{2GM}{c^2}$, G is the gravitational constant, M is the mass of a massive central body, v is speed of the test particle relative to the central body, and r is distance of the test particle from the central body (Wagener, 2007).

The energy, E of a particle can be derived from the Lagrangian:

$$E = m_0 c^2 \frac{e^{R/r}}{\gamma^2} = mc^2 e^{R/r} \quad (27)$$

noting that E = -L.

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$, and $m = \frac{m_0}{\gamma^2}$. m is also useful to define a variable gravitational mass (Wagener, 2005).

$$E = mc^2 e^{R/r} \quad (28i)$$

Similarly, to the photo-electric effect or for the electromagnetic energy(Wagener, 1987)

$$E_e = \tilde{m}c^2 e^{r_e/r} \quad (28ii)$$

From equation (28i), it was noted that m decreases with velocity for the systems. In the case of a planet, such as mercury, it describes not only an elliptical path for the planet, but also a precession of the ellipse (Wagener 2007). This relation relates the kinematical energy, $\tilde{E}$ of special relativity to the frequency of a photon (Hugh and Roger 2004).

$$\tilde{E} = \tilde{m}c^2 = h\nu \quad (29)$$

$$\tilde{m} = \gamma\, \tilde{m}_0 \quad (30)$$

The quantity, $\tilde{m}$ is the electromagnetic mass of a particle, and it varies with rest mass $\tilde{m}_0$ according to equation (30) (Hasan, 2011).

The above relations for $\tilde{E}$, as well as the link between electromagnetism and the kinematics of special relativity persuade one to associate $\tilde{E}$ with the electromagnetic energy of a system. Therefore, substituting equation (28ii) with equation (30), we obtain:

$$E_e = \tilde{m}_0 c^2 \frac{e^{R/2r}}{\sqrt{1 - v^2/c^2}} = \tilde{m}c^2 e^{R/2r} \quad (31)$$

in terms of ϕ_g, equation (31) become:

$$E_e = \tilde{m}_0 c^2 \frac{e^{\phi_g/c^2}}{\sqrt{1 - v^2/c^2}} = \tilde{E} e^{\phi_g/c^2} \quad (32)$$

where we used the general potential:

$$\phi_g = \frac{GM}{r} = \frac{Rc^2}{2r} \quad (33)$$

Deriving for energy relation for the strong and gravitational interaction:

By applying Maclaurin's series to the energy constant generated from the lagrangian above $(E = mc^2 e^{R/r})$; and taking the first and second terms, we obtain:

$$E = mc^2\left(1 + \frac{R}{r}\right) \quad (34)$$

Factorizing equation (34), noting that:

$exp\left(\frac{r}{R}\right) = \left(\frac{r}{R} + 1\right)$ (Wagener, 2005), we get:

$$E = \frac{mc^2 R}{r} exp\left(\frac{r}{R}\right) \quad (35)$$

Also, repeating using the same procedure for the electromagnetic energy in equation (28ii), we obtained:

$$E_e = \frac{\tilde{m}c^2 r_e}{r} \exp\left(r/r_e\right) \quad (36)$$

where, $\tilde{m}$ is the electromagnetic mass of the particle, r is distance of the test particle from the central body, $r_e = -\frac{e^2}{\tilde{m}_{e0} c^2}$ is the classical electron radius, $r = r_q = |r_e|$; and $\tilde{m}_{e0}$ is the electromagnetic rest mass of the electron. Therefore, equation (36) can be written as:

$$E_e = -\frac{Q^2}{r} \exp\left(-\frac{r}{r_q}\right) \quad (37)$$

where Q^2 is defined as: $Q^2 = \tilde{m}c^2 r_q = \tilde{E} r_q$ (38)

By substituting equation (30) into equation (38),

we obtain: $Q^2 = \gamma \tilde{m}_0 c^2 r_q$ (39)

where Q is some effective charge of strong interaction, $\tilde{m}$ is the mass of the particle (the carrier of interaction), r_q is the radius related to the particle under consideration, E_e is the total electromagnetic electron energy.

The strong and the gravitational interaction potential given in the equation (37) has the form of the Yukawa potential and can be directly placed in the Dirac equation (see equation 25).

$$\left(i\hbar\frac{\partial\psi}{\partial t}\right)_{esg} = \left[c\alpha.\left(\vec{p} - \frac{e}{c}\vec{A}\right) - e\Phi + \beta mc^2 + \sqrt{2EeA_0} - \frac{Q^2}{r} e^{-r/r_q}\right]\psi \quad (40)$$

When r_q is too large, the effective energy comes from the gravitational force. The Dirac equation reduces to the form:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\gamma m_0 c^2 r_q}{r} e^{-r/r_q}\,\psi \quad (41)$$

Taken distance as $r_q \approx r$ and integrating, we obtain the wave function ψ as: $\psi \sim e^{i\gamma m_0 c^2 t/e\hbar}$ (42)

This is the wave character of the gravitational forces. Therefore, the wave function given by equation (42) describes the wave structures of both the gravitational and the strong interactions.

The interaction of the particles for a so much charged, mass concentrated, and highly energetic space can be described by a single equation as:

$$\left(i\hbar\frac{\partial\psi}{\partial t}\right)_{esg} = \left[c\alpha.\left(\vec{p} - \frac{e}{c}\vec{A}\right) - e\Phi + \beta mc^2 + \sqrt{2EeA_0} - 2\frac{Q^2}{r} e^{-r/r_q}\right]\psi \quad (43)$$

where the factor 2 describes the combination of the strong and the gravitational forces for such a space.

Lagrangian of weak local interaction of the charged leptons

Introducing the weak interaction, we then start with lagrangian of weak local interaction. This theory is based on the lagrangian that does not contain the particle masses. The particle masses arise after the introduction of a scaler field which interacts leptons and gauge fields (Hagiwara *et al.*,1993).

The lagrangian of massless left- handed leptons is given by (Murray 1959)

$$l_0(L) = i\sum_l \bar{L}_l \gamma^\mu d_\mu L_l \qquad (44)$$

where $L_l = \binom{v_l}{l}_L \equiv \binom{v_{lL}}{l_L}$, $v_{lL} = \frac{1}{2}(1+\gamma^5)v_l$ and $l_L = \frac{1}{2}(1+\gamma^5)v$

The lagrangian of the weak local interaction of the charged leptons is given by (Pollard 1964) and (Brack 1983):

$$L^{(w)} = 2\sqrt{2}G_f J_\lambda^{(w)} + J_v^{(w)} \qquad (45)$$

and

$$J_v^{(w)} = \frac{1}{2}\left[\bar{e}\gamma_v(1+\gamma^5)v_e + \bar{\mu}\gamma_v(1+\gamma^5)v_\mu + \bar{\tau}\gamma_v(1+\gamma^5)v_\tau\right] \equiv \frac{1}{2}\sum_l \bar{l}\gamma_v(1+\gamma^5)v_l \qquad (46)$$

where G_f is the universal fermi constant and also called the strength coupling of weak interaction, $J_v^{(w)}$ is the four current, $l \equiv e, \mu, \tau$ for electron, muon, tau respectively and $v_l = v_e, v_\mu, v_\tau$ for their neutrino. The virial theorem states that the potential energy of the system is twice the kinetic energy of the system in minus sign (Parker 1954). Since the Lagrangian in classical mechanics is written by:

$$L = T - V \qquad (47)$$

The total energy is: $E = T + V$ (48)

where V, is the virial theorem which is shown as: $V = 2T$ (49)

Substituting equation (49) into (48), we obtain: $E = 3T$ (50)

Also, substituting equation (49) into (47), we obtain: $L = -T$ (51)

Multiplying through by the negative sign, we obtain: $T = -L$ (52)

Therefore, substituting equation (52) into (50) we obtain: $E = -3L$ (53)

Then, substituting equation (45) into equation (53) the total energy of the weak interactions, in terms of the Lagrangian, can be written as (Pollard 1964).

$$E = -3\left(2\sqrt{2G_f J_\lambda^{(w)}} + J_v^{(w)}\right) \qquad (54)$$

where $G_f = \frac{g^2}{m^2}$ (55)

"g" is effective charge of the weak interaction and "m" is the mass of test particle relative to the central body. Incorporating equation (54) into equation (43), we obtain:

$$\left(i\hbar\frac{\partial\psi}{\partial t}\right)_{egsw} = \left[c\alpha.\left(P - \frac{e}{c}A\right) - e\Phi + \beta mc^2 + \sqrt{2EeA_0} - 2\frac{Q^2}{r}e^{-r/r_q} - 3\left(2\sqrt{2G_f J_\lambda^{(w)}} + J_v^{(w)}\right)\right]\psi \qquad (56)$$

as a result, the unified equation of interactions is obtained as equation (56)

The last term in this equation describes the weak interactions with the neutrinos dynamics. The equation (56) includes the weak, electromagnetic, gravitational, neutrinos and the strong interactions where the strong and the gravitational interactions are thought to be unified for the considered space.

IV. DISCUSSION

The unification was conducted using two methods, Statistical method and Analytical method. Statistically, we obtain from the result of the linear regression a good correlation coefficient of 0.6. (Figure 1). Equation 6 suggestively indicates that gravitational and electromagnetic interactions relate; noting that F_g is gravitational interaction, U_{mag} is the electromagnetic interactions and $\psi = 0.74$ is the slope of the plot.

Analytically, unification of the four fundamental forces was obtained by introducing Dirac equation as a strong candidate for unification process. The energy relation of strong and the gravitational interaction, was derived (equation 36). Therefore, the derivation governs the gravitational and strong interaction (known as Yukawa potential). Therefore, the wave function as given by equation (42) describes the wave structures of both the gravitational and the strong interactions and was put into the Dirac equation for unification.

In order for the motion of the system to be bounded, the virial theorem was introduced and was written in the relativistic form. Therefore, the virial theorem has to be used to convert the lagrangian of the weak interactions to the form of energy (equation 54); and the unified equation of interactions was derived as equation (56) to describe the true universe.

In comparison, Wagener (2009) limited its scope on unifying the three fundamental forces: Gravitation, Electromagnetic and the strong force with the help of the derivation of the Yukawa potential, without the Dirac equation of unification. In this work, attempt was made to unify the four fundamental forces with the use of classical electron radius into the strong nuclear interaction and the virial theorem into the lagrangian of the weak nuclear interaction for a proper bound of the unification into the Dirac equation as a strong candidate of unification.

V. CONCLUSION

At unification of fundamental forces, the high energy structure of the universe needs the validity of the four fundamental forces. As a result, the Dirac equation is a strong candidate for unification of the four fundamental forces. In addition, the unification process obtained from the extragalactic source for gravitational and electromagnetic interaction, accounted for a good correlation coefficient which indicates validity of the both interaction.

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