

A New Proposed PDF for the Sub –Optimum Receiver Architecture

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Abstract— The detection performance of communication systems in general is limited by the presence of undesirable energy in the received signal. And this undesirable energy at communication receiver is modeled as the sum of gaussian noise and impulsive interference for which closed form probability density function generally does not exist. Due to this implementation of optimum receivers becomes very difficult. In this paper an alternate PDF is proposed written in closed form which provides a much simple architecture.

Index Terms— Alpha-stable, interference, receiver empirical characteristic function, sub-optimal..

I. INTRODUCTION

In **communication systems**, **noise** is an error or undesired random disturbance of a useful information signal. The **noise** is a summation of unwanted or disturbing energy from natural and sometimes man-made sources. These unwanted energy arise from a variety of sources which may be considered in one of two categories.

- (1) Interference, usually from a human source .
- (2) Naturally occurring random noise

Noise is generally modeled as AWGN, in which case closed form, optimum receivers exist and are well known. On the other hand, interference is generally impulsive in nature and hence cannot be modeled as AWGN.

'Additive white Gaussian noise (AWGN) is a basic noise model used in Information theory to mimic the effect of many random

processes that occur in nature. The modifiers denote specific characteristics:

- **Additive** because it is added to any noise that might be intrinsic to the information system.
- **White** refers to the idea that it has uniform power across the frequency band for the information system. It is an analogy to the color white which has uniform emissions at all frequencies in the visible spectrum.
- **Gaussian** because it has a normal distribution in the time domain with an average time domain value of zero.

In communications and electronics, **interference** is anything which modifies, or disrupts a signal as it travels along a channel between a source and a receiver. The term typically refers to the addition of unwanted signals to a useful signal. **Interference** is generated by other signals (in other circuits or, more likely, in the same circuit), so it's artificial noise. Also the signal itself can generate interference, for instance if there is a conflict between subsequent symbols, or not perfect matching on a transmission line. **Noise** is everything that is not useful signal, so can be due to interference, temperature, impurities, gamma rays, moon phase or whatever. So interference is noise but the inverse is not true.

As, we already now interference is generally impulsive in nature and hence cannot be modeled as AWGN. Instead, impulsive and heavy tail distributions such as Cauchy, Gaussian mixture and most notably, the α -stable family of distributions have been proposed in the paper. Closed form pdfs generally do not exist for the α -stable case, hence only sub-optimal receivers are possible. This article focuses only on inference modeled by α -stable distributions (which includes Cauchy distribution as a special case).

Assuming interference is the only source of noise contamination, than several sub-optimal receivers have been already proposed for the α -stable case. But In practical conditions however, often both

noise and interference are present at the same time in varying degrees. The pdf of the resultant mixture then is neither purely Gaussian nor α -stable. In this paper we consider α -stable distributed interference contaminated by AWGN.

The contribution of this paper is an alternate model for the pdf motivated by the intuitive fact that the resultant mixture of AWGN and S α S noise would also necessarily have a distribution with a heavy-tail similar to the associated S α S component.

II. NOISE MODEL

The characteristic function of a general α -stable random variable X is given as

$$\phi_X(\omega) = \exp\{j\alpha\omega - \gamma|\omega|^\alpha [1 + j\beta\text{sign}(\omega)\zeta(\omega, \alpha)]\}$$

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Where,

$$\zeta(\omega, \alpha) = \begin{cases} \tan(\frac{\alpha\pi}{2}) & \text{for } \alpha \neq 1 \\ \frac{2}{\pi} \ln |\omega| & \text{for } \alpha = 1, \end{cases}$$

Consider the noise random variable v given by

$$v = n_s + n_g$$

Here, n_s is a symmetric α -stable (SaS) distributed random variable, and n_g is the random gaussian variable with variance $\sigma_g^2 = 2\gamma_g$ is assumed that n_s and n_g are independent random variables.

The characteristic function of the sum of these independent random variable is

$$\phi_V(\omega) = \phi_s(\omega)\phi_g(\omega) = e^{-\gamma_s|\omega|^\alpha} e^{-\gamma_g\omega^2}$$

Thus, the pdf of noise may now be written as

$$f_V(v) = \frac{1}{\pi} \int_0^\infty e^{-\gamma_s|\omega|^\alpha} e^{-\gamma_g\omega^2} \cos(\omega v) d\omega.$$

For the evaluation of PDF for real time statistical applications, numerical integration is often not a practical option.

To circumvent this problem, in this paper we propose an empirically motivated alternate pdf composed from the sum of a Gaussian term representing the central lobe and a slowly-decaying Paretian tail.

Before presenting the alternate pdf, here we will review the heavy tail model of an symmetric α -stable distribution. For a standard α -stable ($\alpha < 2$) random variable X with dispersion γ ,

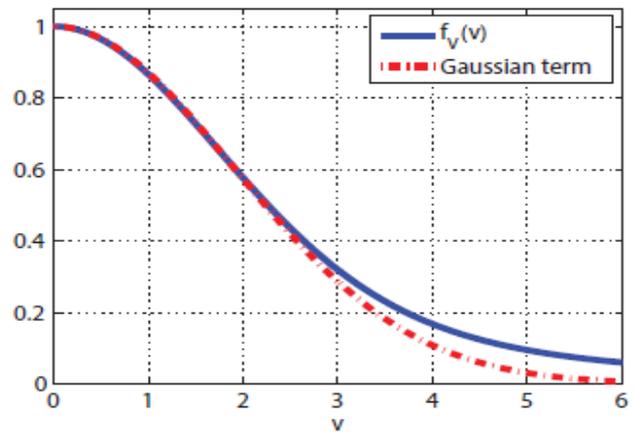
$$\lim_{x \rightarrow \infty} P(X > x) = \frac{\gamma C_\alpha}{x^\alpha}$$

Differentiating above equation results in,

$$S_\alpha(\gamma, 0, 0) \sim \frac{\alpha\gamma C_\alpha}{x^{\alpha+1}}, \quad x \rightarrow \infty$$

which shows that asymptotically (here \sim “asymptotically varies as”), the SaS density function decays proportional to $x^{-(1+\alpha)}$.

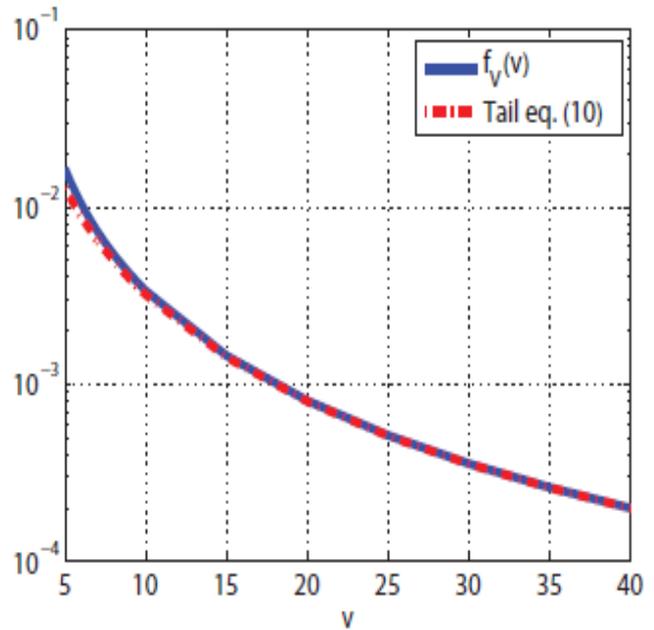
Now we are taking a example, comparing the Gaussian component, the tail component and proposed pdf with exact pdf of v for $\alpha = 1, \gamma_s = 1, \gamma_g = 1$



(a) Central lobe

Figure 1

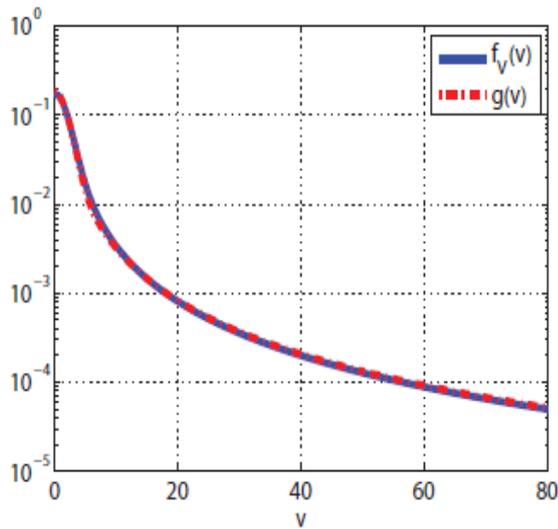
Fig. 1a shows a comparison of the central lobe of (5) (normalized to maximum value of 1) with a Gaussian term $e^{-x^2/4\gamma_g}$. The tail of the Gaussian term decreases to zero quickly, which suggests that the Gaussian term above may serve as a good approximation to the central lobe of $f_V(v)$.



(b) Tail

Figure 2

Fig.1b shows a comparison of the asymptotic tail of (5) with the asymptotic tail as predicted by (12) on a log scale. Here, we can easily observed that the tails essentially overlap, that leads to a possible conclusion that asymptotically the tail of (5) can be represented by (12). From this example, it may be concluded that a sum of appropriately scaled Gaussian term and the asymptotic tail term should lead to a reasonable approximation of $f_V(v)$.



(c) $f_V(v)$ vs $g(v)$

Figure 3

The alternate pdf in the most general terms may now be represented as,

$$g(v) = \frac{1}{I} \left(c_1 g_0 e^{-\frac{v^2}{4\gamma_{sg}}} + \frac{\hat{\alpha}\hat{\gamma}_s C_{\hat{\alpha}}}{|v|^{\hat{\alpha}+1} + c_2} \right)$$

where c_1 , c_2 and γ_{sg} are parameters to be estimated and the terms g_0 and I are defined as follows:-

For appropriate scaling of the Gaussian term, $f_V(0)$ can be approximated by using Cauchy-Schwarz inequality and a correction factor to replace the inequality by an approximate equality,

$$f_V(0) \approx g_0 = \frac{c_0}{\pi} \sqrt{\left(\int_0^{\infty} e^{-2\hat{\gamma}_s \omega^{\hat{\alpha}}} d\omega \right) \left(\int_0^{\infty} e^{-2\hat{\gamma}_g \omega^2} d\omega \right)}$$

$$I = 2c_1 g_0 \sqrt{\pi \gamma_{sg}} + \frac{2(1-c_1)g_0 \Gamma\left(\frac{\hat{\alpha}}{1+\hat{\alpha}}\right) \Gamma\left(1 + \frac{1}{1+\hat{\alpha}}\right)}{\left((1-c_1)g_0 / (\hat{\alpha}\hat{\gamma}_s C_{\hat{\alpha}})\right)^{1/(\hat{\alpha}+1)}}$$

which is a constant for a given set parameters.

CONCLUSION

In this paper, we proposed an alternate, PDF for the case of AWGN channel which is further contaminated with symmetric α -stable interference, which can be implemented to make sub-optimum receiver implementation easy.

All simulations were performed using Matlab running on a machine with dual core, 2.53GHz CPU with 4GB RAM

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