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Maxwell's Electrodynamics in Curved Space-Time

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***Abstract*—This article discusses some of the issues concerning the interaction of material particles with electromagnetic radiation. The investigation is based on solution of the Maxwell-Einstein equations. Basically four issues were explored: (i) A contribution in the ponderomotive force acting onto the probe particle which is determined by the curvature of space-time metric induced by the spherical electromagnetic wave; (ii) A metric which corresponds to the gravitational field created together by a massive source and an electromagnetic wave; (iii) A stability of the electromagnetic vacuum near space-time horizons; (iv) A real topology of space-time. The two last questions involve the non-wave solutions of the Maxwell-Einstein equations. We discuss also the loss of information accompanying the process of transformation a converging spherical electromagnetic wave into a diverging one.**

***Index Terms*— Ponderomotive force, Electromagnetic wave, Space-time metric, Gravitation, Black hole, Instanton.**

I. Introduction

In general case investigation of electromagnetic phenomena in curved space-time needs to solve the system of associated Maxwell-Einstein (ME) equations:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi K}{c^4} T_{ik} ; F^{ik}_{,k} + \Gamma^l_{kl} F^{ik} = 0 \qquad (1)$$

here R – is a trace of the Ricci tensor R^i_k: $R = R^i_i$, g_{ik} – is the metric tensor; T_{ik} and F^{ik}– are tensors of energy-momentum and electromagnetic one; Γ^i_{kl} – are the Kristoffel symbols; c – is the light speed in vacuum, K – is the gravitation constant; indices i, k, l have values 0, 1, 2, 3; on repeated indices assumes summation; comma means usual, not covariant derivative [1].

The tensor T^{ik} and the Kristoffel symbols Γ^i_{kl} has the form [1]

$$\begin{aligned} T^{ik} &= \frac{1}{4\pi}\left(-F^{il} F^k_l + \frac{1}{4} g^{ik} F_{lm} F^{lm} \right) \\ \Gamma^i_{kl} &= \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right) \end{aligned} \qquad (2)$$

Equations (1) describe the mutual influence of material and electromagnetic fields at each other. Less interesting cases corresponding to the given material fields need to be addressed separately second group of equations by using the material field as a parameter.

This article discusses some of the issues of interaction of material particles and electromagnetic fields in frames of the two described approaches.

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II. Ponderomotive force acting from the electromagnetic wave onto the probe particle

We have investigated solutions (1), corresponding to the presence of spherical electromagnetic waves (SEMW) at spatial infinity. As is known, a massive particle placed in the field of electromagnetic wave (EMW), feels the so-called ponderomotive force, acting from the field of EMW which consists of several components [2], having a various nature. The following discussion focuses on the force exerted by a spherical electromagnetic wave on a massive particle, which according to tradition, too, can be called ponderomotive. The origin of it is due to the curvature of space-time caused by the field of EMW.

As shown in the articles [3, 4] the space-time metric, which is induced by a spherical electromagnetic wave, is characterized by an interval

$$\begin{aligned} ds^2 &= g_{00} c^2 dt^2 + g_{11} dr^2 - r^2 (d\theta^2 + \sin^2\theta \cdot d\varphi^2) \\ g_{00} &= -g_{11}^{-1} = 1 - \frac{r_c}{r} + \left(\frac{r_s}{r}\right)^2, r_c = \frac{l(l+1)c}{\omega}, r_s^2 = \frac{K}{2c^4} G^2 \end{aligned} \qquad (3)$$

Here ω –is the frequency of an EMW, l – is the orbital moment momentum of an EMW, G – its amplitude. This metric was found by the solution of the coupled Maxwell-Einstein (ME) equations (1). It has spherical symmetry despite that the source in the right side of the Einstein equation – energy-momentum tensor of the field of the EMW has no spherical symmetry. It is due to the deflection of light rays [1, 5] from their straight propagation what eliminates the non-spherical features in the field of the EMW [6]. This leads to that for large $l \geq 3$ the observed field of spherical EMW becomes spherically symmetric, losing its dependence on angle coordinates [6]. If we take into account that deflection mentioned above must be counted twice this inequality must be replaced to $l \geq 2$ (Zayko Y.N., unpublished).

Due to the difference in values $r_c \sim 10^{-5}$ sm (the order of the wavelength of light) and $r_s \sim 10^{-16}$ sm (for solar radiation) [4] at distances of the order of the wavelength of light in the expression for the $g_{00} = e^{-\alpha}$ (conventional notation in the literature for the g_{00}) one can restrict only by the first term. Let us compare it with the expression for the g_{00} of the metric near a massive charged body [5]

$$g_{00} = 1 - \frac{r_g}{r} + \frac{r_q^2}{r^2}, r_g = \frac{2KM}{c^2}, r_q^2 = \frac{q^2 K}{4\pi\varepsilon_0 c^4} \qquad (4)$$

r_g – is a Schwarzschild radius, M and q– are the mass and charge of a point-like particle located at the $r = 0$. We see that the ponderomotive action of the field of a spherical electromagnetic wave onto the massive probe particle placed at a distance r from the focus of EMW is equivalent with the gravitational effect on it from a mass $M=\frac{l(l+1)c^3}{2K\omega}$ placed in the focus of EMW. For the optical wavelength range ~ 10^{-5} sm it amounts to $M\sim l(l+1)\cdot 10^{22}$ g.

In addition, this mass has a charge $q=\sqrt{4\pi\varepsilon_0}G$. This leads to the conclusion that the rigorous solution of the problem requires a different approach, including the self-interaction of electromagnetic waves, which can be done only within the frames of non-Abelian gauge theories [7]. Nonetheless, given the difference in the scales it can be argued that at distances of the order of the wavelength of light the effects of self-interaction can be neglected and to use the results of the articles [3, 4].

III. Metric Induced by the Spherical EMW and the Massive Source

Let us consider more general problem of finding a space-time metric due to common action of the spherical EMW and a point-like particle of mass m placed in its focus. It is believed that due to the nonlinearity of the Einstein equations the general action of gravity sources cannot be reduced to the sum of each of them individually. Consider this question in detail. In the article [4] it was shown that the Maxwell-Einstein (ME) equations under consideration can be written in the two equivalent forms (due to supposition that $g_{00}=-g_{11}^{-1}$)

$$
\begin{aligned}
&-e^{-\alpha}\left(\frac{1}{r^2}-\frac{\alpha'}{r}\right)+\frac{1}{r^2}=\frac{r_s^2}{r^4}\\
&\frac{1}{2}e^{-\alpha}\left[\alpha''-(\alpha')^2+\frac{2\alpha'}{r}\right]=-\frac{r_s^2}{r^4}
\end{aligned}
\tag{5}
$$

Primes mean the derivatives on r. These really are non-linear equations. But if we introduce the variable $t=r_s/r$, than for the function $y(t)=e^{-\alpha}=g_{00}$ we get, respectively, the equations

$$
\begin{aligned}
&ty'-y=t^2-1\\
&y''=2
\end{aligned}
\tag{6}
$$

(under the assumption of constant wave amplitude G) where the second equation is obtained by differentiating the first one. Their solution has the form $y(t)=t^2+C_1t+C_2$. If we choose the constants equal to $C_1=-r_c/r_s, C_2=1$ then we receive the expression (3) for the metric. The equations (6) are linear and at the distances $r\sim r_c$, the second term in the metric ~ $(r_s/r_c)^2$ << 1 can be omitted, and the summation of the effects of the gravitational sources is permissible.

We apply this reasoning to the case of an EMW behavior near black hole. We use the results of the articles [3, 4] and [8] on the structure of solutions of the Einstein-Maxwell equations with point-like mass as a source. At the distances $r_c < r < \infty$ they remain valid also in this problem (there is a limit on the mass of the black hole $m<M$, arising from the condition $r_g < r_c$). As shown in these papers, Einstein-Maxwell equations, together with the wave-type solutions have also instanton- like solution that binds converging and diverging EMW and thereby deprive the black hole of it brutal role of the "gravedigger" of information. To show this we select the TM- and TE-waves for the coding the bits of information. Their characters, as well as a character of metric stay unchanged during the transition from converging to diverging wave. This leads to conservation of the bit of information coded by them.

IV. Instantons of the ME-Equations and Stability of the Electromagnetic Vacuum

The curvature of space-time associated with spherical EMW leads to a restructuring of the electromagnetic vacuum. Both converging and diverging EMW exist in different vacuums, transition between which is carried out with the help of the instanton [4]. There is no mathematical proof of this assertion today. We can, however, raise the question of stability of the electromagnetic vacuum near the event horizons for the metrics (3) or (4).

Consider the classical action for the electromagnetic field [1]

$$
S=-\frac{1}{16\pi c}\int F_{ik}F^{ik}\sqrt{-g}\,dx^0dx^1dx^2dx^3 \tag{7}
$$

where g – is the determinant of the quantities g_{ik}. As was shown in the article [3] the quantity S (7) for the case of the spherical EMW of TM-type can be expressed by one component of electromagnetic tensor $F_{01}=\Psi(r,t)P_l(\theta)$, where $P_l(\theta)$ – is a Legendre polynomial of order l. Finally, (7) can be written as (P_l^1 - associated Legendre polynomial)

$$
S=-\frac{1}{8\pi c}\int r^2drdx^0d\Omega\left\{\begin{array}{l}\left[e^{\alpha}\left(\frac{\partial f}{\partial x^0}\right)^2-e^{-\alpha}\left(\frac{\partial f}{\partial\rho}\right)^2\right]\\ \frac{\left[P_l^1(\theta)\right]^2}{\left[l(l+1)\right]^2}-\frac{f^2}{r^4}\left[P_l(\theta)\right]^2\end{array}\right\} \tag{8}
$$

$$
d\Omega=\sin\theta d\theta d\varphi,\ f=r^2\Psi
$$

Variable ρ is determined by the equation $d\rho=e^{\alpha}dr$. Performing integrating on the angular coordinates, one can rewrite (8) in the form

$$
S=-\frac{1}{2(l+1)c}\int d\rho dx^0\left\{\begin{array}{l}\left[\left(\frac{\partial f}{\partial x^0}\right)^2-\left(\frac{\partial f}{\partial\rho}\right)^2\right]\cdot\\ \frac{1}{l(l+1)}-V(\rho)\end{array}\right\} \tag{9}
$$

$$
d\rho=e^{\alpha}dr,V(\rho)=e^{-\alpha(\rho)}\frac{f^2}{r^2(\rho)}
$$

The expression in the curly brackets (up to a factor), is the density of the Lagrangian of the electromagnetic field of spherical wave with amplitude $f(x^0,\rho)$, and $V(f, r(\rho))$ - is the density of the potential energy. The behavior of $\partial^2V/\partial f^2$ can give an information on the nature of the stability of the electromagnetic field vacuum. Usually vacuum instability is considered in the sense of the birth of pairs of particles [9], what can be taken into account in (7) in the first order in the Plank constant $\hbar$. Here, the term "instability" is understood in a different sense, namely in terms of stability (or instability) of the wave-type solutions of field equations. It can be shown that the inclusion of the said amendment does not change final conclusions. Taking for $e^{-\alpha}$ the expressions (3) or (4), where for simplicity we restrict ourselves by only first terms we conclude that for $r > r_c$ (or $r > r_g$) electromagnetic waves are stable, and for $r < r_c$ (or $r < r_g$)- they aren't, what is the cause of arising of non-wave solutions of Maxwell's equations, which have been in the works [3, 4, 8] identified with instantons.

V. Instantons of the Maxwell-Einstein equations and space-time topology

In recent years, increased interest in the study of topology of real space in connection with the proof in 2003 by G. Perelman the Poincare conjecture [10] which was formulated by him in 1904, the essence of which is that if an arbitrary one-dimensional closed curve in the n - dimensional manifold can be shrunk to a point, then this manifold is homeomorphic to the n - dimensional sphere (has a topology of a sphere).

Numerous popular expositions (see, for example, [11]) on the subject, make attempts to apply this mathematical result to real space and "prove" the fact which is considered obvious, namely, that our universe resembles a sphere, and not, for example, a torus. At the same time, in a manner typical of mathematicians, the question of the physical implementation of such evidence is not considered. Rope, which is appeared in [11], is no more than a metaphor.

In the report [12] various physical ways to check this result were discussed, and it was shown that they all are untenable - in all cases the methods implemented with the help of physical objects (light rays) cannot be used to achieve target for reasons not associated with the topology.

The report [12] investigated the question of topology by using another homotopic mapping - sphere into a sphere. According to the articles [3, 4, 8], front of a converging spherical EMW cannot be shrunk to a point. This question is closely connected with the question of focusing of spherical EMW. As mentioned above, converging EMW cannot classically be transformed into diverging EMW. This has less to do with the singularity of solutions of the ME equations, how with the possibility of capture of some light rays in the metrics (3) or (4) [4]. The conversion process goes quantum mechanically through an intermediate state - instanton which is due to tunneling connects wave solutions of the original equations ME, which are realized near different vacuums.

Homotopy, which is referred to above relates to the contraction of the front of spherical electromagnetic wave, what may be done only till the distances of the order of the instanton size. Inability to shrink the front further is conditioned by the fact that at shorter distances waves do not exist. Although this fact has no direct relationship to the Poincare theorem, it is also associated with the topology of space. This result indicates that the physical space is not homeomorphic sphere, and has a more complex topology.

Another indication of this is a violation by instantons of ME equations the so-called "weak energy condition" $T_{\alpha\beta}\xi^{\alpha}\xi^{\beta} > 0$ [8], where $T_{\alpha\beta}$ – is an energy-momentum tensor of electromagnetic field, and ξ – is any non-space-like 4 – vector [13]. As is known, this condition is associated with the requirement of the absence of so-called "wormholes" [14], representing the topological features of space-time.

VI. Conclusions

In this section we will briefly summarize the results of the present paper. They are classified so as in the abstract.

(i)The ponderomotive action of the field of a spherical electromagnetic wave onto the massive probe particle is equivalent the gravitational action of a massive and charged source located in the focus of the wave;

(ii) At distances of the order of length of light wave the partial contributions in the resulting space-time metric caused separately by the massive source and by the spherical electromagnetic waves are summed;

(iii) It is shown that when passing through the horizon of space-time metric the stability of the electromagnetic vacuum is disrupted in the sense that the wave solutions of Maxwell-Einstein equations become unstable and the solution of non-wave type - instanton is occurring;

(iv) Instantons of the Maxwell-Einstein equations affect the character of topology of space-time.

The last question has been widely discussed in the literature [15, 16, 17].

The results of this article is directly related to the problem of photon falling onto a black hole. That problem long time was in the center of the debate about the fate of the information in the vicinity of a black hole [18, 19], about the results of which was fascinatingly told in the book of one of the participants of the discussion [20]. One of the results of the present work is the conclusion that the panelists did not take into account all the factors that effect on its outcome.

Nonetheless, the loss of information will take place in this description, too. It is connected with the non-wave nature of the instanton, what doesn't permit to store information from the region where instanton occupies. If we take into account that spherical EMW are the main transfer agents of information from the Past to the Future [21] then we conclude that this transfer goes only with the probability $P < 1$ [4, 8]. So we can say that the process of evolution of physical systems in time is a non-deterministic.

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